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Carpooling with heterogeneous users in the bottleneck model

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ABSTRACT

We investigate the effects of carpooling in a dynamic equilibrium model of congestion, which captures various dimensions of heterogeneity: heterogeneity in the preference for and cost of carpooling, heterogeneity in values of time and values of schedule delay. We investigate various policy scenarios: no-toll, first-best pricing, and subsidization of carpooling. The optimally differentiated subsidy equals each type of users' marginal external benefit (MEB) of switching to m -person carpooling or carpooling, which turns out to be heterogeneous for "ratio heterogeneity", where the ratios of the values of time and schedule delay vary, and homogeneous for "proportional heterogeneity", where these values vary in fixed proportion over the population. If such differentiation over users is impossible, the subsidy is a weighted average of the MEB's, with the weights reflecting the relative sensitivity of the group size of carpoolers to the subsidy. Using a numerical example, we investigate the welfare effects and distributional effects of different policies. The relative efficiency of the differentiated subsidization first increases and then falls with the degree of ratio heterogeneity, and decreases over the entire parameter range and more with the degree of proportional heterogeneity.

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1. Introduction

Many cities face increasing traffic flows and road congestion, and certainly so in the morning peak on arterial roads. This forces municipalities to consider alternatives for conventional car use, such as public transport, bicycles and carpooling. Because of its flexibility and the more personal atmosphere than transit, carpooling is an interesting alternative for solo car use (Ferguson (1997), Caulfield (2009)). Recently, with the proliferation of technology-enabled ride matching, carpooling has got more attention than before (Masoud and Jayakrishnan (2017a,b), Wang et al. (2017)). Various policy measures have been proposed to encourage carpooling, including carpool lanes, free carpool parking, ride parking and carpool matching platforms. Also a subsidy may be an effective policy to attract people to switch to carpooling. For example, many ridesharing platforms like Didi, Uber and Meituan-Dianping in China compete to provide subsidies to users, and this made the number of carpoolers soar¹. This raises an interesting and important issue: how to design a subsidization scheme to attract more solo drivers to switch to carpooling and thus increase the social welfare? This question becomes particularly complex when

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¹ <https://www.caixinglobal.com/2018-01-25/didis-new-subsidies-augur-second-price-war-101202995.html> (accessed on 25 January 2018).

commuters are heterogeneous. How does this heterogeneity influence the welfare effects and the distributional effects of subsidization? That is one of the main questions that we will address in this paper.

We define carpooling as the sharing of a car between people on a trip from a certain origin to a specific destination. Monetary cost savings result from sharing, for example, fuel cost, tolls and parking charges. But extra gathering time and inconvenience costs negatively influence the intention to switch to carpooling (e.g., Kocur and Hendrickson 1983). These may differ substantially across travelers. Instead of considering homogeneous preferences for car pooling versus solo driving, we use a logit model where the non-systematic (random) part of utility represents idiosyncratic preferences over the two modes and differences in their costs. Also in the systematic part of utility we allow for heterogeneous preferences, by having differences in values of time and schedule delay. Our model therewith ensures interior equilibria, where some people carpool and some do not, as is also seen in reality.

Several earlier studies have studied carpooling behavior in the morning commute traffic equilibrium. Yang and Huang (1999) use a deterministic equilibrium model to discuss carpooling and optimal congestion pricing in a multi-lane highway with or without HOV (High-Occupancy Vehicle) lanes and find that in the presence of HOV lanes, first-best pricing for a social optimum requires differentiating the toll per vehicle across segregated lanes. When toll differentiation cannot be applied, the optimal uniform toll is a weighted average of the marginal external congestion costs between non-carpooling and carpooling commuters. Huang et al. (2000) present deterministic and stochastic models to investigate the shifting behavior between carpooling and driving alone. They find that carpooling is sensitive to traffic congestion reduction only when a congestion externality-based tolling scheme is implemented. Qian and Zhang (2011) analyze the interactions among transit, driving alone and carpool with identical commuters, addressing that parking availability is another factor stimulating carpooling. Xiao et al. (2016) investigate carpooling behavior with a fixed ridesharing ratio and a parallel transit lane under constrained parking spaces, and find that the best system performance can be realized with joint consideration of total travel cost and vehicle emission cost. Ma and Zhang (2017) formulate an analytical continuous-time dynamic ridesharing problem for a single bottleneck in the morning commute, where the ridesharing ratios and parking charges are time-dependent. To maintain a positive ridesharing rideship, Liu and Li (2017) propose a time-varying compensation scheme for ridesharing users, considering the congestion evolution over time.

These studies usually assume that all commuters are homogeneous. Still, various studies have found that heterogeneity in travel mode and departure timing selection is important, and that heterogeneous commuters may exhibit large behavioral differences in departure time choice, and in response to congestion tolls. Ignoring preference heterogeneity may cause a biased estimation of the impacts of policies. It is, thus, of great importance to incorporate preference heterogeneity.

Dynamic models of peak hour congestion have considered different forms of heterogeneity. In particular when heterogeneity concerns both heterogeneity in value of time and in values of schedule delay, different possibilities arise. We distinguish three ideal types: ratio heterogeneity (de Palma and Lindsey (2002), van den Berg and Verhoef (2011a,b)), proportional heterogeneity (Vickrey (1973), van den Berg and Verhoef (2011b)), and more general heterogeneity (Newell (1987), Lindsey (2004), Börjesson and Kristoffersson (2014), Wu and Huang (2015), Liu et al. (2015), Chen et al. (2015), Li et al. (2017), Takayama and Kuwahara (2017)).² “Ratio heterogeneity” refers to heterogeneity in the ratio of the value of time over the value of schedule delay, or α_i/β_i , in the conventional notation. It reflects the willingness to accept greater schedule delays in order to reduce travel time. It hence measures differences in arrival time flexibility, and could stem from differences in job type, trip purpose, family status and age. “Proportional heterogeneity” refers to the case where the values of time α_i and schedule delay β_i vary over individuals, but in fixed proportions, so that the ratio is the same for everybody. It could stem from differences in incomes. “More general heterogeneity”, finally, occurs when the two types of heterogeneity would jointly lead to an unrestricted bivariate distribution in our paper. We will first consider separate ratio heterogeneity and proportional heterogeneity, and then the more general heterogeneity. We will also extend our model to “ γ heterogeneity”, where the cost of late arrival γ differ across travelers, and other values are the same for all (Arnott et al. (1988, 1994), van den Berg (2014)).

Such heterogeneity affects the welfare gain of policies, and is naturally an important determinant of distributional effects of policies (see, e.g., Arnott et al. (1988), Small and Yan (2001), van den Berg and Verhoef (2011a,b)). These distributional effects are important, if only because they are a major reason for resistance against a new policy. Moreover, if one would like to compensate those who lose disproportionately due to a new policy, one needs to know which types of drivers lose, and by how much. We will see that carpooling imposes a positive externality and allows travelers to share monetary costs, alongside the discomfort and extra travel time to drivers it may cause. The positive externality makes it worthwhile to provide a subsidy, to make carpooling more attractive. We examine three policy schemes: no tolling, first-best tolling, and carpool subsidization. To the best of our knowledge, the effects of subsidization of carpooling on welfare and the distributional effects have not been analyzed before for heterogeneous users with dynamic congestion.

² Apart from these three types of heterogeneity, there are also “ β or γ heterogeneity” (Arnott et al. (1988), Arnott et al. (1994), van den Berg and Verhoef (2014)), which captures differences in the willingness to arrive before or after the preferred arrival time, and “ t^* heterogeneity” (Cohen (1987), Arnott et al. (1988)), which captures the heterogeneity in the preferred arrival time t^* .

Table 1
Nomenclature.

α_i	Value of time (VOT) of type i users.
β_i	Value of schedule delay early of type i users.
γ_i	Value of schedule delay late of type i users.
δ_i	Compound preference parameter of type i users: $\delta_i = \beta_i \gamma_i / (\beta_i + \gamma_i)$.
TT_{ff}	Free-flow travel time.
TT_{ff}^p	Extra time cost of gathering the people together for carpooling.
c_{fuel}	Fuel cost for each car per trip.
θ	Inconvenience cost due to carpooling per carpooler.
s	Bottleneck capacity.
N	Total number of users.
c_i^j	$j = a, p$. Generalized cost for each travel mode of group i .
p_i^j	$j = a, p$. Generalized price for each travel mode of group i .
ϕ_i	The scale of utility of group i .
ϵ_i^j	The stochastic mode preference utility.
U_i^j	$j = a, p$. The random utility with mode j of group i .
N_i^j	$j = a, p$. The number of users with mode j of group i .
F_i^j	$j = a, p$. The fraction of mode j chosen by group i .
F_i^p	The derivative of F_i^p with respect to the subsidy.
F_i^m	The probability of choosing m-person carpooling of group i users.
$F_i^{m(p)}$	The conditional probability of choosing m-person carpooling (given carpooling) of group i .
Δc_i	Change in generalized cost by switching from driving alone to carpooling (per group i user).
Δp_i	Change in generalized price by switching from driving alone to carpooling (per group i user).
τ	Time-varying toll with the first-best tolling.
MEB_i	Marginal external benefits of group i incurred by switching to carpooling.
S_i^*	Optimal second-best subsidy of group i with social welfare maximization, $i = H, L$.
S^*	Optimal third-best subsidy with social welfare maximization.
cs_i	Consumer surplus per user of group i .
$cs^\#$	Arbitrary constant of the integration in consumer surplus.
Δcs_i	The change of consumer surplus per user due to the introduction of carpooling.
SW	Social welfare.
ω	Relative efficiency.
t^*	Preferred arrival time.

This paper studies carpooling in the bottleneck model with various dimensions of heterogeneity.³ This paper makes four main contributions to the literature. *First*, we investigate the marginal external benefits (MEB) of users by switching from solo driving to carpooling or m-person carpooling, under different types of heterogeneity. The MEBs turn out to be heterogeneous for ratio heterogeneity and homogeneous for proportional heterogeneity. For γ heterogeneity, the MEBs are homogeneous for the early arrival users and heterogeneous for the late arrival users. *Second*, we derive the closed-form optimal subsidies on carpooling, maximizing the social welfare. Given subsidization scheme, the optimal subsidy should be set to equal the marginal external benefit of each type's users, or the weighted average of the MEB's, with the weights reflecting the relative sensitivity of the group size of carpoolers to the subsidy. *Third*, we investigate the welfare effects and distributional effects. Our study shows that the introduction of voluntary carpooling itself makes all users better off, making it a politically attractive option. Nevertheless, with more proportional heterogeneity, the group with the high value of schedule delay benefits more for all pricing schemes. With more ratio heterogeneity, when tolling is implemented, the group with the low value of time benefits more. The relative efficiency of the carpool subsidization decreases with the degree of proportional heterogeneity, and first increases and then decreases for most of the range with the degree of ratio heterogeneity. *Finally*, heterogeneity in preferences for and costs of carpooling are incorporated in our model, and allows for interior equilibria with each type choosing both solo driving and carpooling with positive probability. Naturally, our results confirm that first-best tolling and second-best subsidization could help enhance carpooling.

The remainder of this paper is organized as follows. Section 2 explains the model set-up and the equilibria under homogeneity. Section 3 introduces ratio heterogeneity, proportional heterogeneity and the combination of these two with two types of users. Section 4 develops a numerical example, and provides sensitivity analyse. Section 5 extends the model to multiple-person carpooling, carpooling with more discrete types of users, and heterogeneity in travel delay late. Section 6 concludes.

For ease of reference, Table 1 below summarizes the notation. The notation will also be introduced in the text.

³ We only consider car travel and not also public transport or mobility services such as taxis or Uber (e.g., Djavadian and Chow (2017), Wang et al. (2017), Masoud and Jayakrishnan (2017a)). We only consider a single road, there are no HOV lanes (e.g., Yang and Huang (1999)) and no parking (e.g., Xiao et al. (2016), Ma and Zhang (2017)).

2. The model

2.1. Set-up

We begin our exposition with listing our assumptions, and in passing, introducing the notation. Every morning, N commuters travel from home to a workplace connected by a single road that is subject to bottleneck congestion. We assume that the total number of drivers N is fixed. We ignore other transport modes such as biking. Everybody travels by car; either solo or in a 2-person carpool.

Travel time cost equals travel time multiplied by the value of time (VOT). The VOT of type i users is denoted α_i . The travel time is the sum of the delay from queuing at the bottleneck and the free-flow travel time. The delay from queuing by mode j is denoted TT^j ($j = a, p$), and equals the number of cars in the queue when entering it, divided by the capacity of the bottleneck. As a carpool has two persons in it, carpooling raises the effective capacity of the bottleneck, when expressed in passengers per unit of time. For driving alone, the free-flow travel time is TT_{ff} ; for carpoolers, it is $(TT_{ff} + TT_{ff}^p)$, where TT_{ff}^p is the extra time cost of gathering the people together for pooling, which is assumed to be equal for the two carpoolers. In the analysis the free flow travel time TT_{ff} is normalized to 0; the numerical study will consider a positive value.

A person's bottleneck cost equals schedule delay cost plus queuing time cost. The schedule delay cost is the cost due to arriving at a different time than the most preferred arrival moment, t^* , which is assumed to be identical for all and is normalized to 0. We follow [Small \(1982\)](#) and [Arnott et al. \(1993\)](#), and use schedule delay costs that are linear in the time difference between t^* and the actual arrival time, t . The shadow cost of type i users per hour for arrivals earlier than t^* is β_i ; for hours late it is γ_i . The schedule delay cost thus is $\max[-\beta_i t, \gamma_i t]$.

Riding with a stranger decreases the comfort and privacy for the drivers. Therefore, carpoolers inevitably undergo average inconvenience cost θ , which is assumed to be a fixed amount per trip and per person. This adds to the time loss TT_{ff}^p introduced earlier. The fuel cost per trip, c_{fuel} , is also assumed to be a fixed amount per trip. For carpoolers, the drivers will share the fuel cost equally.

The travel cost is the sum of the bottleneck cost and the free-flow travel time cost. The generalised travel cost per trip per person of type i users is then the sum of the travel cost, the fuel cost and the inconvenience cost:

$$\begin{aligned} c_i^a[t] &= \max(-\beta_i t, \gamma_i t) + \alpha_i TT^a[t] + c_{fuel} && \text{when driving alone;} \\ c_i^p[t] &= \max(-\beta_i t, \gamma_i t) + \alpha_i TT^p[t] + \alpha_i TT_{ff}^p + c_{fuel}/2 + \theta && \text{when carpooling;} \end{aligned} \quad (1)$$

where superscript a indicates driving alone, and superscript p indicates carpooling.

In equilibrium, for both modes, the travel cost for a specific group of users needs to be constant over the arrival times used by those drivers. We use the random utility maximization, representing unobserved idiosyncratic preferences and costs for carpooling versus driving alone, to characterize the discrete choice behavior between driving alone and carpooling. The random utility function of individual from type i with mode j is:

$$U_i^j = -c_i^j + \frac{\epsilon_i^j}{\phi_i}, \quad j = a, p, \quad (2)$$

where U_i^j depends on a deterministic utility component $-c_i^j$, and on a stochastic idiosyncratic mode preference utility ϵ_i^j , which is assumed to be identically and independently distributed. This unobserved ϵ_i^j reflects the differences in how costly carpooling is compared to normal driving and differences in preferences for carpooling. The parameter ϕ_i defines the scale of systematic utility of type i users. The larger ϕ_i , the less important idiosyncratic preferences and costs, and hence the more deterministic the choices are.

When these idiosyncratic preferences are i.i.d. Gumbel distributed, in equilibrium, the mode choice probabilities F_i^j are governed by the following logit formulae:

$$F_i^a = \frac{e^{-\phi_i c_i^a}}{e^{-\phi_i c_i^a} + e^{-\phi_i c_i^p}}, \quad F_i^p = \frac{e^{-\phi_i c_i^p}}{e^{-\phi_i c_i^a} + e^{-\phi_i c_i^p}}. \quad (3)$$

We use the familiar log-sum formula ([Train \(2009\)](#)) to express the consumer surplus, which is given by the expectation of Eq. (2) over the two alternatives,

$$cs = \sum_i \frac{\log(e^{-\phi_i c_i^a} + e^{-\phi_i c_i^p}) + cs^\#}{\phi_i}, \quad (4)$$

where $cs^\#$ is the arbitrary constant of the integration that represents the fact that the absolute level of utility cannot be measured. This constant is irrelevant from a policy perspective.

The heterogeneity in the scale of utility only changes the separate probabilities for each type and does not change the properties of the model (see Eqs. (3)–(4)). We hence focus on homogeneous scale of utility in most of the analysis.

Each individual needs to decide on a departure time from home to minimize the total travel cost of the entire car. In doing so, she makes a trade off between travel time cost and schedule delay cost. Equilibrium is achieved when no individual can reduce her travel cost by altering the departure time. The three subsections look at three cases, namely the

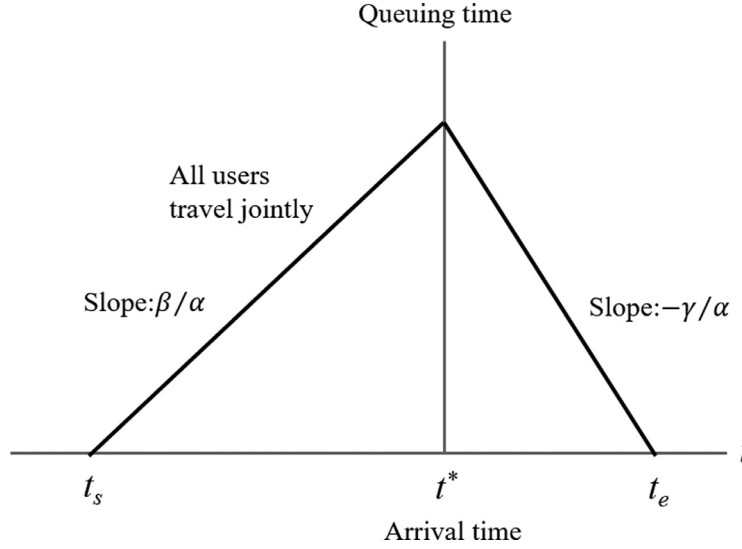


Fig. 1. No-toll equilibrium with homogeneity.

no-toll equilibrium (NT), the first-best equilibrium (FB), and the second-best subsidization on carpooling (SB). It is instructive to start our exposition with the homogeneous preferences model, as the effects of carpooling are easier to understand in this case.

2.2. No-toll equilibrium with homogeneous users

In the dynamic no-toll equilibrium with homogeneous users but with carpooling (NT), the generalized travel cost should be constant over time as long as arrivals occur. Fig. 1 illustrates the equilibrium. The solid line shows the queuing times. It indicates that the carpoolers and solo drivers are traveling jointly over the peak as the two groups have the same ratio β/α or γ/α per vehicle: both α and β in a carpool are twice that of a solo-drive car (see Arnott et al. (1988), van den Berg and Verhoef (2011a,b)).

The per person equilibrium costs for driving alone and carpooling are:

$$\begin{aligned} c_{NT}^a &= \delta \frac{N_{NT}^a + N_{NT}^p/2}{s} + c_{fuel}, \\ c_{NT}^p &= \delta \frac{N_{NT}^a + N_{NT}^p/2}{s} + \alpha TT_{ff}^p + c_{fuel}/2 + \theta, \end{aligned} \quad (5)$$

where δ is a composite preference parameter: $\delta = \beta\gamma/(\beta + \gamma)$. N_{NT}^a and N_{NT}^p denote the number of solo drivers and carpoolers. The bottleneck costs (the first terms in Eq. (5)) are straightforward to derive from the conventional bottleneck model, where these costs are equal to $\delta N/s$ (e.g., Arnott et al. (1987)).

A user who switches from driving alone to carpooling will experience a change in travel cost of Δc_{NT} :

$$\Delta c_{NT} = c_{NT}^a - c_{NT}^p = -\alpha TT_{ff}^p - \theta + c_{fuel}/2. \quad (6)$$

While α can differ between drivers, θ and c_{fuel} are assumed identical across drivers. A positive value of Δc_{NT} reflects a positive incentive to form a carpool.

Using the logit model, the equilibrium number of carpoolers and solo drivers are:

$$N_{NT}^p = \frac{N}{1 + e^{\phi(\alpha TT_{ff}^p + \theta - c_{fuel}/2)}}, \quad N_{NT}^a = \frac{N}{1 + e^{\phi(-\alpha TT_{ff}^p - \theta + c_{fuel}/2)}}, \quad (7)$$

where N denote the total number of users. Eqs. (6)–(7) show that with deterministic preferences, a corner solution with only carpooling or solo driving would prevail, as the cost difference is independent of the number of solo drivers and carpoolers. With random utility, choice probabilities for users to choose carpooling and driving alone are both positive.

2.3. First-best equilibrium with homogeneous users

Now we turn to the first-best case (FB), where tolling removes all queuing (Arnott et al. (1987)). This requires the toll to increase at a rate β for early arrivals, and to fall at a rate γ for late arrivals, since this ensures that zero travel time delays constitute the dynamic equilibrium. Because there are two persons in each carpool, the total value of schedule delays in

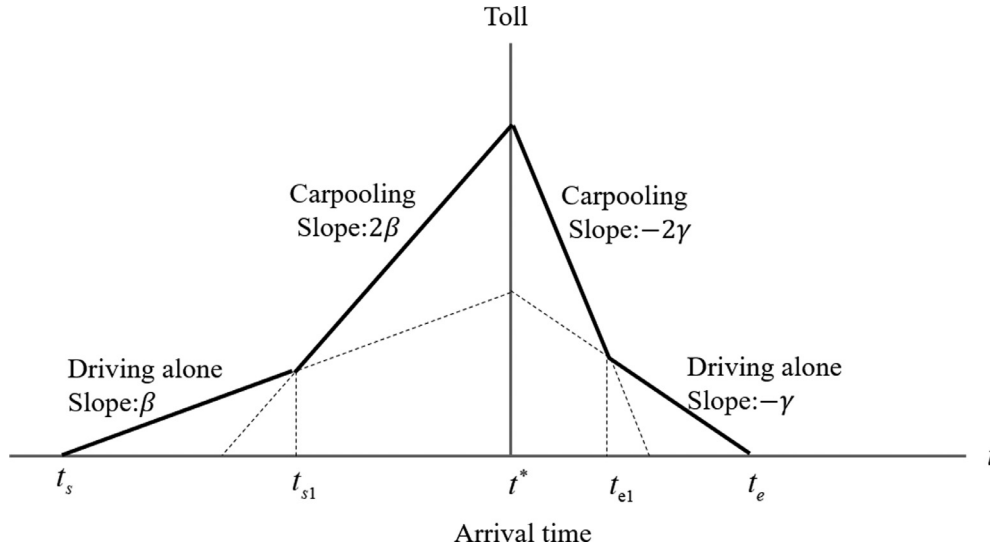


Fig. 2. First-best equilibrium with homogeneity.

a carpool is twice as large as with solo driving. Therefore the toll should also grow or shrink at a double rate to maintain equilibrium. As a consequence, the carpools travel in the center of the peak and the solo drivers travel away from the center: carpools will find an arrival moment closer to t^* more attractive than arriving at moments where solo drivers arrive, since the gain in schedule delay cost outweighs the increase in toll. The opposite applies for solo drivers in the time window where carpools arrive: they would prefer arriving further from t^* as the toll savings exceed the additional schedule delay cost. Hence the temporal separation of traffic that is shown in Fig. 2 illustrates this optimum. The generalized price, when arriving at t , now includes generalized travel cost and the toll. For the carpools, the toll is equally shared by the two travellers in the car. We find the following prices:

$$p_{FB}^a = \delta \frac{N_{FB}^a + N_{FB}^p/2}{s} + c_{fuel}, \quad (8)$$

$$p_{FB}^p = \delta \frac{N}{2s} + \alpha TT_{ff}^p + c_{fuel}/2 + \theta. \quad (9)$$

Carpoolers share the toll by the two drivers, and the separation of types over time means that carpools benefit from the relatively flat toll scheme for solo drivers (see also van den Berg and Verhoef (2011a,b)). At the same time, they suffer from the fact that solo drivers impose a higher demand, per traveler, on the bottleneck capacity. Consequently, for the carpools, there is a $\frac{\delta N_{FB}^a}{2s}$ toll reduction due to sharing in Eq. (9), compared to the case where the same amount of cars would have been occupied by carpools.

For a user who switches from driving alone to carpooling, the price now changes by:

$$\Delta p_{FB} = p_{FB}^a - p_{FB}^p = \delta N_{FB}^a/(2s) - \alpha TT_{ff}^p + c_{fuel}/2 - \theta. \quad (10)$$

Again, a positive value reflects an advantage for carpooling and vice versa.

The number of carpools and solo drivers are respectively:

$$N_{FB}^p = \frac{N}{1 + e^{-\phi \Delta p_{FB}}}; N_{FB}^a = \frac{N}{1 + e^{\phi \Delta p_{FB}}}. \quad (11)$$

Eq. (10) implies that Δp_{FB} is determined by the number of solo drivers N_{FB}^a , while N_{FB}^a is in turn determined by the price difference Δp_{FB} through Eq. (11). Therefore, we cannot obtain analytical solutions of N_{FB}^a and p_{FB}^a . Social welfare in the FB equilibrium is again the sum of consumer surplus and the toll revenues.

2.4. Second-best subsidization with homogeneous users

We now turn to an interesting second-best policy (SB): a flat time-invariant subsidy for carpooling. The generalized cost of driving alone follows the same expression as for the NT case, while for carpooling, a fixed subsidy S is subtracted. For that reason, and because the number of solo drivers and carpools will change, equilibrium costs and prices will change as well. The two groups, however, keep traveling jointly in time. We optimize the subsidy by maximizing social welfare (SW) with respect to S . Note that S is per passenger in a carpool; the subsidy per carpool is therefore $2S$.

The generalized prices for driving alone and carpooling are given by:

$$\begin{aligned} p_{SB}^a &= \delta \frac{N_{SB}^a + N_{SB}^p/2}{s} + c_{fuel}, \\ p_{SB}^p &= \delta \frac{N_{SB}^a + N_{SB}^p/2}{s} + \alpha TT_{ff}^p + c_{fuel}/2 + \theta - S, \end{aligned} \quad (12)$$

reflecting that the type of equilibrium will qualitatively resemble the one depicted in Fig. 1, although the equilibrium share of carpoolers will be different.

To find the second-best subsidy, we maximize social welfare SW , which is the total consumer surplus minus the total subsidy, where the number of carpoolers N_{SB}^p and solo drivers N_{SB}^a are determined by the logit model. Combining Eqs. (3)–(4) and Eq. (12), we derive the social welfare:

$$\begin{aligned} SW &= \frac{N \log(e^{-\phi p_{SB}^p} + e^{-\phi p_{SB}^a}) + Ncs^\#}{\phi} - SN_{SB}^p \\ &= \frac{N \log\left(1 + e^{\phi(-\alpha TT_{ff}^p - \theta + c_{fuel}/2 + S)}\right) + Ncs^\#}{\phi} + \frac{N(\delta N/2s - S)}{1 + e^{\phi(\alpha TT_{ff}^p + \theta - c_{fuel}/2 - S)}} - \frac{\delta N^2}{s} - Nc_{fuel}. \end{aligned} \quad (13)$$

Solving the first-order condition of Eq. (13) yields the optimal subsidy:

$$S^* = \frac{\delta N}{2s}. \quad (14)$$

Each user, irrespective of whether she is a carpooler or a solo driver, benefits with $\frac{1}{2} \frac{\delta}{s}$ decrease in price when one marginal traveller switches to carpooling. The optimal subsidy in Eq. (14) is thus naturally interpreted as the reduction in total social cost following a marginal change from solo-driving to carpooling, on top of the change in cost for that marginal driver. This benefit is therefore external to the choice of an individual, and thus the subsidy equals the marginal external benefit (MEB).⁴ Naturally, with homogeneous users, the MEB is also homogeneous.

3. Heterogeneity

Now we turn to heterogeneous users. When considering heterogeneity, we first assume that users are separated into two discrete groups, which we can denote for each type of heterogeneity as type high (H) and type low (L), where the exact interpretation differs between the cases. To simplify, we assume up-front that carpoolers share the carpool with the same type; i.e., the H type share with the H type and the L type share with the L type. This is, however, consistent with the idea that joint optimization with someone who has the same preferences leads to a better outcome than with someone who has partly conflicting preferences. Therefore, this assumption is not harmful, and furthermore not essential for our results, but it helps in restricting the number of groups traveling on the road to four: solo drivers of H type, solo drivers of L type, carpoolers of H type, and carpoolers of L type.

For ease of exposition and explanation, and development of intuition, we will therefore first be considering two types of heterogeneity: ratio heterogeneity and proportional heterogeneity, and next consider the combination of these two. With ratio heterogeneity, the groups differ because their ratios of value of time over values of schedule delay differ. With proportional heterogeneity, these ratios are the same between groups, but the values themselves differ.

3.1. Ratio heterogeneity

Ratio heterogeneity means heterogeneity in the value of time α , while the schedule delay β and γ are the same for all. The ratio for group i is $\mu_i = \alpha_i/\beta$ (Arnott et al. (1987), van den Berg and Verhoef (2011a)). Users with a high ratio are less willing to queue (or, alternatively, they are more willing to adjust when to arrive), as a higher travel time is relatively more costly for them than a lower schedule delay. The High group has a higher ratio $\mu_H = \alpha_H/\beta$, and the Low group has a lower ratio $\mu_L = \alpha_L/\beta$.

3.1.1. No-toll equilibrium with ratio heterogeneity

The no-toll equilibrium requires travel times by arrival time to grow at a rate $1/\mu_i = \beta/\alpha_i$ when group i arrives ($i = H, L$). Travelers with a high μ_H will choose to arrive relatively early or late, to avoid long travel times. The reverse applies to the L group, which leads to separate traveling: group L arrives closest to t^* , and group H arrives further from t^* . Due to the same ratios applying to solo drivers and carpools within a group, solo drivers and carpools of the same type will travel jointly. There is hence also an incentive over users to only carpool with their own type.

⁴ Instead of maximizing with respect to S , we could also have maximized with respect to the number of solo drivers and carpoolers as in Huang et al. (2000).

Following van den Berg and Verhoef (2011a,b), the group-specific generalized travel costs for driving alone and carpooling are:

$$\begin{aligned} c_H^a &= \delta \frac{N_H^a + N_H^p/2 + N_L^a + N_L^p/2}{s} + c_{fuel}, \\ c_H^p &= \delta \frac{N_H^a + N_H^p/2 + N_L^a + N_L^p/2}{s} + c_{fuel}/2 + \theta + \alpha_H TT_{ff}^p, \\ c_L^a &= \delta \frac{\alpha_L}{\alpha_H} \frac{N_H^a + N_H^p/2}{s} + \delta \frac{N_L^a + N_L^p/2}{s} + c_{fuel}, \\ c_L^p &= \delta \frac{\alpha_L}{\alpha_H} \frac{N_H^a + N_H^p/2}{s} + \delta \frac{N_L^a + N_L^p/2}{s} + \alpha_L TT_{ff}^p + c_{fuel}/2 + \theta, \end{aligned} \quad (15)$$

where the subscript H denotes H-type users and subscript L denotes L-type users. N_i^j denotes the number of i -type travelers with mode j . As these values are endogenous, ratio heterogeneity affects the generalized cost of both types for a given total number of travelers. The higher a user's VOT is, relative to the values of schedule delay, the less queuing this user causes, and the lower this user's congestion externality. Therefore, the bottleneck cost for the L group of each travel pattern is lower than what it would have been if all H-type drivers were replaced by the L-type, and also lower than that of the H group.

For a user who switches from driving alone to carpooling, the travel cost drops by:

$$\Delta c_H = c_H^a - c_H^p = -\alpha_H TT_{ff}^p - \theta + c_{fuel}/2, \quad \Delta c_L = c_L^a - c_L^p = -\alpha_L TT_{ff}^p - \theta + c_{fuel}/2. \quad (16)$$

As α_H is larger than α_L , the price drop between driving alone and carpooling for the H group is lower than that for the L group. Carpooling is thus more attractive for the users with low value of time, reflecting the lower penalty from additional time lost in forming a carpool.

Using the logit model, we can next determine the number of solo drivers and carpoolers for each type users as:

$$N_i^a = \frac{N_i}{1 + e^{\phi(-\alpha_i TT_{ff}^p - \theta + c_{fuel}/2)}}, \quad N_i^p = \frac{N_i}{1 + e^{\phi(\alpha_i TT_{ff}^p + \theta - c_{fuel}/2)}}, \quad (i = L, H), \quad (17)$$

where N_i is the number of type i drivers.

Switching to carpooling imposes a positive externality by decreasing the travel time of other drivers. We find the MEBs by using the total cost function TC and taking the difference between the marginal social cost and the privately incurred cost from switching (see Appendix A):

$$MEB_H = c_H^p - c_H^a - \left(\frac{\partial TC}{\partial N_H^p} - \frac{\partial TC}{\partial N_H^a} \right) = \frac{\delta N_H}{2s} + \frac{\delta \alpha_L N_L}{\alpha_H 2s}, \quad (18)$$

$$MEB_L = c_L^p - c_L^a - \left(\frac{\partial TC}{\partial N_L^p} - \frac{\partial TC}{\partial N_L^a} \right) = \frac{\delta N}{2s}, \quad (19)$$

where the total cost is $TC = N_H^a c_H^a + N_H^p c_H^p + N_L^a c_L^a + N_L^p c_L^p$.

With ratio heterogeneity, the marginal external benefits for users with low value of time exceed that with high value of time, i.e. $MEB_L > MEB_H$.

3.1.2. First-best equilibrium with ratio heterogeneity

First-best tolling removes all the queuing and this requires the toll to increase at a rate β for early arrivals and decrease at a rate γ for late arrivals (de Palma and Lindsey (2002)). Early travelers are ordered by increasing values of β , for the same sort of self-selection mechanism as described before. The carpoolers arrive in the center of the peak due to the doubled value of β , induced by having two persons in each carpool. Because there is no difference of the values of schedule delay between H type and L type, solo drivers from different types travel jointly, and carpoolers from different types travel jointly, as was the case under homogeneity.

In equilibrium, the group-specific generalized prices are:

$$\begin{aligned} p_i^a &= \delta \frac{N_L^a + N_L^p/2 + N_H^a + N_H^p/2}{s} + c_{fuel}, \\ p_i^p &= \delta \frac{N}{2s} + \alpha_i TT_{ff}^p + c_{fuel}/2 + \theta, \quad i = H, L. \end{aligned} \quad (20)$$

The expression for the FB price of solo drivers of both types replicates the price expression for H-type users in the NT situation. However, because the numbers of solo drivers and carpoolers are endogenous, the FB price may still be expected to be different from that in the NT case.

For a user who switches from driving alone to carpooling, the generalized price drops by:

$$\Delta p_i = p_i^a - p_i^p = \delta \frac{N_L^a + N_H^a}{2s} - \alpha_i TT_{ff}^p + c_{fuel}/2 - \theta, \quad i = H, L. \quad (21)$$

3.1.3. Second-best subsidization with ratio heterogeneity

A flat subsidy policy maximizes social welfare, which is again the total consumer surplus minus the total subsidy, by finding the optimal subsidy. In equilibrium, the generalized prices of solo drivers have the same expressions as those for the NT case (see Eq. (15)). The generalized prices for carpoolers are lower than those in the NT case, due to the subsidy. The generalized price reduction, from switching to carpooling is increased by the subsidy, or the price increase is decreased. We first consider the case where the subsidy on carpooling can be differentiated between the two groups, which are respectively denoted by S_H and S_L .

The numbers of carpoolers and solo drivers for both types of drivers are given by:

$$N_i^a = \frac{N_i}{1 + e^{\phi(-\alpha_i TT_{ff}^p - \theta + c_{fuel}/2 + S_i)}}, \quad N_i^p = \frac{N_i}{1 + e^{\phi(\alpha_i TT_{ff}^p + \theta - c_{fuel}/2 - S_i)}}, \quad (i = L, H). \quad (22)$$

Solving the first-order conditions of the social welfare with respect to S_H and S_L yields the optimal second-best subsidy⁵ (see Appendix B):

$$S_H^* = \frac{\delta N_H}{2s} + \frac{\delta \alpha_L}{\alpha_H} \frac{N_L}{2s}, \quad S_L^* = \frac{\delta N}{2s}. \quad (23)$$

The optimal subsidies in Eq. (23) equal the marginal external benefits (MEB) of switching to carpooling for both groups, in Eqs. (18)–(19). This is consistent with what was found for homogeneous drivers. We can also find that S_H^* decreases with α_H/α_L , i.e., the reduction in total social cost from solo-driving to carpooling by H-type users decreases with the degree of ratio heterogeneity.

3.1.4. Third-best subsidization with ratio heterogeneity

It may very well be impossible to give different types of users a different subsidy. When the subsidy cannot be differentiated, (i.e. $S_H = S_L$), to maximize social welfare, a third-best (TB) subsidy S^* can be derived as a weighted average of the MEB's. It amounts to (see Appendix C):

$$S^* = \lambda_H \left(\frac{\delta N_H}{2s} + \frac{\delta \alpha_L}{\alpha_H} \frac{N_L}{2s} \right) + \lambda_L \frac{\delta N}{2s}, \quad (24)$$

with

$$\lambda_H = \frac{F'_H N_H}{F'_H N_H + F'_L N_L}, \quad \lambda_L = \frac{F'_L N_L}{F'_H N_H + F'_L N_L} = 1 - \lambda_H, \quad (25)$$

where F'_i is the derivative of the fraction of i -type carpoolers F_i^p with respect to S . The weights λ_H and λ_L hence depend on the numbers of users of both types and their demand sensitivity to the subsidy. In this condition and for given N 's, the optimal subsidy S^* is between S_H^* and S_L^* , i.e., $MEB_H < S^* < MEB_L$. As it cannot be solved analytically, the specific solution will be demonstrated by numerical examples in Section 4. We define the differentiated subsidy in Eq. (23) as second-best subsidization and the undifferentiated subsidy in Eq. (24) as third-best subsidization.

3.2. Proportional heterogeneity

Now we turn to proportional heterogeneity. This refers to the heterogeneity where the ratio of values of time and schedule delay (α_i/β_i or α_i/γ_i) is fixed, but all values vary in fixed proportions following the scalar k_i : $\alpha_i = k_i \alpha$, $\beta_i = k_i \beta$ and $\gamma_i = k_i \gamma$, ($i = L, H, k_H > k_L$). This type of heterogeneity may well stem from income differences, with apart from the marginal utility of income otherwise identical preferences: all three values α , β , γ depend linearly on the inverse of the marginal utility of income, which decreases with income.

3.2.1. No-toll equilibrium with proportional heterogeneity

Without tolling, travel times follow the same pattern as with homogeneity: all users travel jointly. This is because the ratios β_i/α_i and γ_i/α_i measure the willingness to queue, and hence determine the arrival order of drivers. Under proportional heterogeneity, these ratios are the same for all users.

The following costs levels apply:

$$c_i^a = \delta_i \frac{N_H^a + N_H^p/2 + N_L^a + N_L^p/2}{s} + c_{fuel};$$

$$c_i^p = \delta_i \frac{N_H^a + N_H^p/2 + N_L^a + N_L^p/2}{s} + c_{fuel}/2 + \theta + \alpha_i TT_{ff}^p. \quad i = H, L. \quad (26)$$

⁵ The first-order and second-order conditions of maximization hold for an interior solution and there is one unique optimum. We have also tested the expressions in Eqs. (23) and (24) numerically by maximizing the social welfare, and the results are consistent with the analytical subsidies.

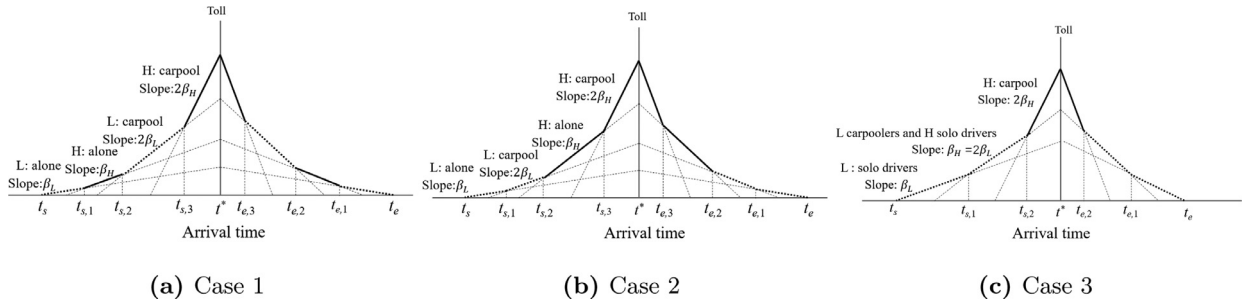


Fig. 3. First-best equilibrium with proportional heterogeneity.

For a user who switches from driving alone to carpooling, the travel cost decreases by:

$$\Delta c_i = c_i^a - c_i^p = -\alpha_i T T_{ff}^p - \theta + c_{fuel}/2, \quad i = H, L. \quad (27)$$

The higher α_i is, the smaller the cost difference in favor of carpooling. Using the same logic as in the previous section, we find the marginal external benefits due to switching to carpooling for each type are:

$$\begin{aligned} MEB_H &= c_H^p - c_H^a - \left(\frac{\partial TC}{\partial N_H^p} - \frac{\partial TC}{\partial N_H^a} \right) = \frac{\delta_H N_H + \delta_L N_L}{2s}, \\ MEB_L &= c_L^p - c_L^a - \left(\frac{\partial TC}{\partial N_L^p} - \frac{\partial TC}{\partial N_L^a} \right) = \frac{\delta_H N_H + \delta_L N_L}{2s}, \end{aligned} \quad (28)$$

where TC is the total cost; N_H^a, N_H^p, N_L^a and N_L^p are determined by the logit model.

With proportional heterogeneity, the marginal external benefits are therefore the same for all users.

3.2.2. First-best equilibrium with proportional heterogeneity

Queuing is again a pure loss, and the first-best toll eliminates it. Each commuter chooses her departure time and travel mode. Types will then arrive in order of their β_i . The drivers with the highest values choose to arrive closest to the preferred arrival time, as for them schedule delays are most costly and they are thus most willing to pay the toll to avoid these. The L-type arrives the furthest from t^* , as they care least about schedule delays. The first-best toll thus fully separates the types by self-selection of drivers, and it not only removes the queuing but also reduces total schedule delay cost. We still assume that carpoolers share the carpool with the same type, so that the compound travel delay for each carpool car should be $2\beta_i$, ($i = L, H$). The assumption of $\alpha_H > \alpha_L$ ensures $\beta_H > \beta_L$ holds. The arrival order before t^* contains 3 cases, based on the relative values of β_H and $2\beta_L$.

- Case 1: if $\beta_H < 2\beta_L$, the early arrival order is: L-type solo drivers, H-type solo drivers, L-type carpoolers, H-type carpoolers.
- Case 2: if $\beta_H > 2\beta_L$, the early arrival order is: L-type solo drivers, L-type carpoolers, H-type solo drivers, H-type carpoolers.
- Case 3: if $\beta_H = 2\beta_L$, the early arrival order is: L-type solo drivers, mixed L-type carpoles and H-type solo drivers, H-type carpoolers.

The first-best equilibrium prices can again be written as the schedule delay cost at the moment that the relevant isoprice line intersects the horizontal axis. Fig. 3 illustrates the equilibrium of the above three cases. As the cost difference in the logit model is related to the number of solo drivers and carpoolers of each type, the generalized prices and numbers of solo drivers and carpoolers have no closed-form solutions. van den Berg and Verhoef (2011b) showed that with only proportional heterogeneity, first-best tolling reduces the generalized price (i.e. toll plus travel costs) for all users, except for those with the very lowest values, who are unaffected. This also means that the gain of first-best tolling increases with the degree of proportional heterogeneity. We will use a numerical example in Section 4 to illustrate that this result also applies in the current context.

3.2.3. Second-best subsidization with proportional heterogeneity

As noted, with proportional heterogeneity the MEB is the same for all, and as we will see, hence so is the optimal carpool subsidy.

The government maximizes social welfare:

$$SW = \frac{N_H \log(e^{-\phi p_H^a} + e^{-\phi p_H^p}) + N_L \log(e^{-\phi p_L^a} + e^{-\phi p_L^p}) + N c_s^\#}{\phi} - S_L N_L^p - S_H N_H^p, \quad (29)$$

where $p_i^a = c_i^a$, $p_i^p = c_i^p - S_i$ by Eq. (26), N_i^a and N_i^p are derived from the logit model.

Solving the first-order conditions of SW with respect to S_H and S_L respectively, we then obtain a uniform subsidy:⁶

$$S^* = S_H^* = S_L^* = \frac{\delta_H N_H + \delta_L N_L}{2s}. \quad (30)$$

The uniform subsidy is consistent with the marginal external benefit in Eq. (28). The optimal subsidy thus again equals the marginal external benefit from switching to carpooling. The third-best undifferentiated subsidy in this case replicates second-best subsidization.

3.3. Generalizing the model: both ratio and proportional heterogeneity

Now we turn to the full setting, in which the combination of ratio and proportional heterogeneity brings changes in the carpooling behavior and welfare effects. For convenience, we assume that each type has a different ratio $\mu_i = \alpha_i/\beta_i$ and different values of β_i and γ_i , so that all groups travel separated in time for all possible equilibria. We denote the labels H and L such that $\alpha_H > \alpha_L$. We furthermore impose a common ratio $\eta_i = \gamma_i/\beta_i = \eta$ for both types, to ensure symmetry across groups. This assumption is not essential, but it helps in restricting heterogeneity of values of time and schedule delay in two dimensions: μ and β . The assumption means that the timing of the peak is independent of the heterogeneity. The effects will prove to be a combination of those in the previous two subsections.

In the NT and SB equilibrium, the early arrival orders are determined by the ratio of μ_i , which is the same for solo drivers vs carpoolers to type i drivers. Consequently, the arrivals of solo drivers and carpoolers of each type are mixed. In contrast to the earlier discussion of separate ratio heterogeneity where $\alpha_H/\beta > \alpha_L/\beta$, and proportional heterogeneity, for which $\alpha_H/\beta_H = \alpha_L/\beta_L$, there are now the two possibilities of $\alpha_H/\beta_H \geq \alpha_L/\beta_L$ and $\alpha_H/\beta_H < \alpha_L/\beta_L$. When $\alpha_H/\beta_H > \alpha_L/\beta_L$, L-type users arrive at the center of peak hours, where the opposite occurs when $\alpha_H/\beta_H < \alpha_L/\beta_L$, i.e., H-type users arrive at the center of peak hours. We will not consider the case of $\alpha_H/\beta_H = \alpha_L/\beta_L$, as it reduces to the separate proportional heterogeneity.

Similarly, with the first-best tolling, the early arrival order is determined by β_i and $2\beta_i$. Because β_H can now be lower than β_L despite $\alpha_H > \alpha_L$, the possible early arrival order before t^* can be separated into 4 cases. Apart from the Case 1, Case 2 and Case 3 defined in Section 3.2.2, a new case happens when $\beta_H < \beta_L < 2\beta_H < 2\beta_L$. We denote it as Case 0, where the early arrival order is H-type solo drivers, L-type solo drivers, H-type carpoolers, L-type carpoolers.

We use a numerical example to illustrate the results of this case in Section 4, where the NT equilibrium is given analytically, and the first-best and subsidization equilibrium are solved numerically.

4. Numerical example

This section presents numerical results to illustrate the models developed above. We consider ratio heterogeneity, proportional heterogeneity and the full case. The differences between the types of heterogeneity will turn to be important. We will find that the introduction of carpooling makes all users better off, but a gap remains between different drivers' benefits from different policies.

For the first-best tolling, we use the Method of Successive Average (MSA)⁷ to find the equilibrium. After discussing the base cases, we then investigate heterogeneity and its impacts on efficiency. Apart from welfare gains, policy makers are also interested in the distributional effects of policies. We therefore also investigate heterogeneity and distributional effects. For simplicity, we there only consider ratio heterogeneity and proportional heterogeneity. After this, we turn to the sensitivity analyses. The outcomes are sensitive to the parameterizations, and hence it is important to present these results.

4.1. Calibration of the numerical models

As in van den Berg and Verhoef (2011a,b), we use $N = 9000$ and $s = 3600$. We consider a trip of 30 km with a free-flow travel time TT_{ff} of 30 minutes and an extra travel time for carpooling TT_{ff}^p of 6 minutes. Fuel costs per trip c_{fuel} , are € 7.30. The inconvenience costs for carpooling θ are supposed to be € 4 per trip. Our base value of the utility scale ϕ is set to be 1. As the constant $cs^\#$ in consumer surplus does not influence the travel mode behavior, we arbitrarily set $cs^\# = € 10$. We suppose H-type and L-type users are evenly distributed, i.e., $N_H = N_L = 4500$.

For homogeneity, we use a VOT α of 10 (van den Berg (2014)). The schedule delay parameters follow the ratios $\beta/\alpha = 39/64$ and $\gamma/\alpha = 1521/640$, established in Small (1982).⁸ For ratio heterogeneity, to make sure the average value of time is 10, we use $\alpha_H = 12.50$, and $\alpha_L = 7.50$. For proportional heterogeneity, we use $k_H = 1.2$ and $k_L = 0.8$. Hence, $\beta_L = 4.87$, $\beta_H =$

⁶ The first-order and second-order conditions of maximization holds for an interior solution and there is one unique optimum.

⁷ The Method of Successive Average (MSA): Step 1, Initialize. Calculate the initial free-flow travel price, $p_{i(0)}^a, p_{i(0)}^p, i = H, L$. set $n=0$; Step 2, Calculate the augmented flow with the logit model, $N_{i(n+1)}^a = \frac{e^{-p_{i(n)}^a}}{e^{-p_{i(n)}^a} + e^{-p_{i(n)}^p}} N_i$, $N_{i(n+1)}^p = \frac{e^{-p_{i(n)}^p}}{e^{-p_{i(n)}^a} + e^{-p_{i(n)}^p}} N_i$; Step 3, Use MSA to update the flow, where $N_{i(n+1)}^a = (1 - \frac{1}{n}) N_{i(n)}^a + \frac{1}{n} N_{i(n+1)}^a$ and $N_{i(n+1)}^p = (1 - \frac{1}{n}) N_{i(n)}^p + \frac{1}{n} N_{i(n+1)}^p$; Step 4, Terminate check. If $|N_{i(n+1)}^j - N_{i(n)}^j| < 10^{-6}, j = a, p$, terminate and output the optimal solution $N_{i(n+1)}^j$. Otherwise, set $n = n + 1$ and go to step 2.

⁸ Under these parameters, the relative efficiency of the carpool subsidization with homogeneous users is 0.46.

Table 2

Outcomes under the base case with heterogeneous drivers.

	Ratio				Proportional			Full			
	NT	FB	SB	TB	NT	FB	SB	NT	FB	SB	TB
Fraction of carpoolers H	0.17	0.61	0.96	0.98	0.18	0.75	0.99	0.17	0.52	0.93	0.97
Fraction of carpoolers L	0.25	0.72	0.99	0.99	0.24	0.56	0.99	0.25	0.80	0.99	0.98
Price of driving alone H	24.40	21.65	19.74	19.73	26.33	22.01	20.64	22.83	20.50	18.94	18.87
Price of carpooling H	26.00	21.21	16.50	16.02	27.88	20.91	16.14	24.43	20.43	16.39	15.51
Price of driving alone L	19.68	19.15	15.99	15.99	19.99	17.83	16.19	19.34	18.88	15.86	15.83
Price of carpooling L	20.78	18.21	11.03	11.78	21.14	17.60	11.28	20.44	17.48	11.26	11.97
Optimal subsidy H	-	-	4.85	5.31	-	-	6.06	-	-	4.15	4.96
Optimal subsidy L	-	-	6.06	5.31	-	-	6.06	-	-	5.70	4.96
$\Delta cs_H^{\#}$	1.45	4.96	9.21	9.68	1.70	4.80	11.71	1.27	4.14	7.60	8.44
$\Delta cs_L^{\#}$	1.35	5.29	9.73	8.98	1.28	3.98	9.72	1.30	11.40	9.13	8.43
Toll revenue (*10,000)	-	3.04	-4.81	-4.69	-	2.81	-5.40	-	2.68	-4.27	-4.35
Social welfare (SW) (*10,000)	-10.63	-5.33	-8.17	-8.18	-11.63	-5.13	-8.74	-9.76	-4.98	-7.68	-7.69
Relative efficiency* (ω)	0	1	0.46	0.46	0	1	0.44	0	1	0.43	0.43

Note: $\Delta cs_i^{\#}$ is the consumer surplus change from no carpooling to with carpooling; ω is the welfare gain of a policy relative to the NT policy divided by the gain of FB policy relative to the NT policy.

7.31, $2\beta_L = 9.74$, $2\beta_H = 14.62$. The early arrival order under first-best pricing is therefore: L-type solo drivers, H-type solo drivers, L-type carpoolers, H-type carpoolers (Case 1).

For the full case, we use $\alpha_H = 12.5$, $\alpha_L/\alpha_H = 0.6$, $\alpha_H/\beta_H = 2.4$, $\alpha_L/\beta_L = 1.2$, $\gamma_L/\beta_L = \gamma_H/\beta_H = 1521/390$. Thus $\alpha_H = 12.50$, $\alpha_L = 7.50$, $\beta_H = 5.21$, $\beta_L = 6.25$, $\gamma_H = 20.28$, $\gamma_L = 24.38$. In the no-toll and subsidization equilibrium, L-type users arrive in the peak and H-type users arrive in the shoulder. In the FB equilibrium, as $\beta_H < \beta_L < 2\beta_H < 2\beta_L$ holds, the early arrival order is H-type solo drivers, L-type solo drivers, H-type carpoolers, L-type carpoolers (Case 0).

As a benchmark, Δcs in Table 2 indicates that the introducing of carpooling raises the consumer surplus. Even in the NT case, users are better off with than without carpooling. Of course, users can never be worse off than before, as they can always all choose to stick to driving alone and benefit from other drivers forming carpools. In addition, under third-best subsidization and first-best tolling with proportional heterogeneity, H-type carpoolers benefit more than the L-type. For the third-best subsidization, it is because the subsidy H-type carpoolers obtain exceeds their marginal external benefits (see Eqs. (23)–(25)). For the first-best tolling with proportional heterogeneity, it is because H-type carpoolers arrive in the peak enjoying the elimination of delays, and sharing tolling with the other carpooler with carpooling.

The fraction of L-type carpoolers exceeds that of H-type carpoolers, except for the first-best tolling under proportional heterogeneity. There, the fraction of H-type carpoolers is 19 percent higher than the L-type carpoolers. Because by switching to carpooling, H-type carpoolers can not only drive in the peak enjoying the elimination of delays, but also sharing tolling with the other carpooler.

First-best tolling impacts H and L-type users differently. With ratio and proportional heterogeneity, H-type carpoolers benefit most, followed by the H-type solo drivers, L-type carpoolers and L-type solo drivers. H-type users benefit more because they value time savings more. Carpoolers benefit more because they share the toll between two, while enjoying the same travel time gains. When switching to a two-person carpool, users can cut the congestion toll in half and therefore come out almost even on a time-plus-money basis, given the assumed carpooling inconvenience. For the full setting, L-type carpoolers benefit more than H-type solo drivers due to the calibration of $\beta_L > \beta_H$.

In terms of the second-best subsidization, ratio and proportional heterogeneity also lead to different results. For the former, L-type carpoolers benefit the most since the extra time cost of carpooling is lower while the benefit of queue elimination, especially enjoyed by the H-type, no longer exists. Naturally, carpoolers benefit more because they receive the subsidy, followed by H-type solo drivers and L-type solo drivers. Conversely, with proportional heterogeneity, H-type carpoolers again benefit the most.

Finally, we can find that although second-best and third-best subsidization result in different generalized prices, the welfare gains and the relative efficiency of these two subsidization schemes nearly keep the same. For ratio heterogeneity, the relative efficiency is roughly the same as that under homogeneity (0.46), and a modest 2 percent lower under than proportional heterogeneity. For the full case, it is about 0.43, implying that ignoring heterogeneity may slightly overestimate the rule of the subsidization, but only mildly so.

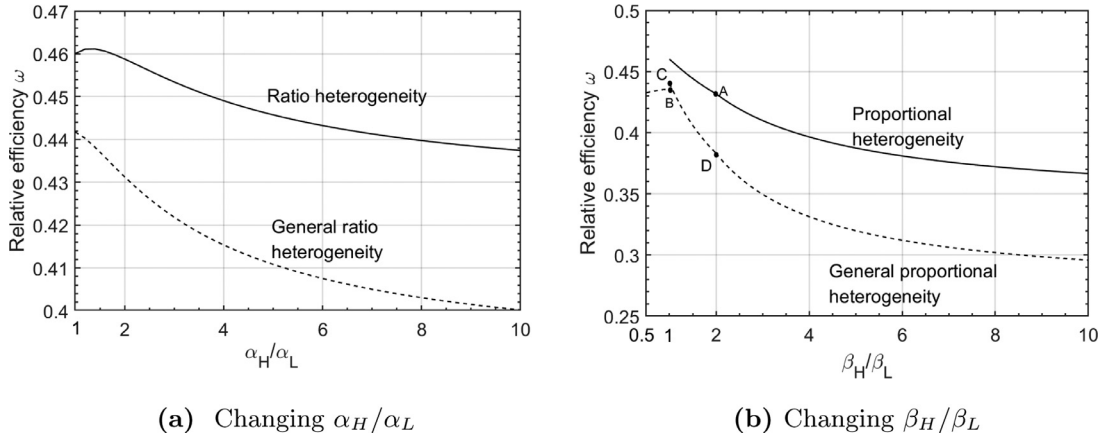
4.2. Heterogeneity and efficiency

As heterogeneity substantially affects the departure timing and hence the welfare effects, we first summarize the possible early arrival order before t^* in our numerical model in Table 3. We consider 3 cases: heterogeneity in α_H/α_L , heterogeneity in β_H/β_L , and the combination of these two. “Separate ratio” means only changing the degree of ratio heterogeneity by varying α_H/α_L , whereas “general ratio” also allows for heterogeneous β_H and β_L ; “separate proportional” means only changing the degree of proportional heterogeneity by varying β_H/β_L , α_H/α_L and γ_H/γ_L by the same percentages, whereas

Table 3Early arrival order before t^* with heterogeneity.

Heterogeneity	NT and SB	FB
Separate ratio	$H^p \& H^a, L^p \& L^a$	$H^a \& L^a, H^p \& L^p$
General ratio	$H^p \& H^a, L^p \& L^a$	H^a, L^a, H^p, L^p
Separate proportional	$H^p \& H^a \& L^p \& L^a$	Case 1, Case 2 or Case 3
General proportional	$H^p \& H^a, L^p \& L^a$	Case 0, Case 1, Case 2
Full	if $\frac{\beta_H}{\alpha_H} > \frac{\beta_L}{\alpha_L}$: $L^a \& L^p, H^a \& H^p$ if $\frac{\beta_H}{\alpha_H} < \frac{\beta_L}{\alpha_L}$: $H^a \& H^p, L^a \& L^p$	or Case 3

Note: The arrival orders in Case 0, Case 1, Case 2 and Case 3 are defined in Section 3.2.2 and 3.3.

**Fig. 4.** Effects of heterogeneity on relative efficiency.

“general proportional” also allows for heterogeneous β_H/α_H and β_L/α_L ; “Full” means the full case by varying α_H/α_L and β_H/β_L together.

Fig. 4 shows what happens to the relative efficiency of the second-best subsidization, where the subsidy can differ over types, when the two groups become more different in terms of value of time and schedule delay; in other words, as the degree of heterogeneity (defined as $\frac{\alpha_H}{\alpha_L}$ or $\frac{\beta_H}{\beta_L}$) increases.⁹ When $\alpha_H/\alpha_L = 1$ or $\beta_H/\beta_L = 1$, the two types are the same. Ratio and proportional heterogeneity result in the same equilibrium, and the relative efficiency stays at 0.46, the same value as with homogeneity. The general case shows a relative efficiency slightly below 0.46 at the starting point, because despite $\alpha_H/\alpha_L = 1$ or $\beta_H/\beta_L = 1$, the other values are not at the same levels as in the homogeneous base case for calibration purpose.¹⁰

Fig. 4 (a) shows that with more heterogeneity in α_H/α_L , ratio heterogeneity leads the relative efficiency of the second-best subsidization to first increase over a small range, and then start to slightly fall. For general ratio heterogeneity, the relative efficiency decreases throughout, and strongly more than under separate ratio heterogeneity. From Eq. (17) and Eqs. (21)–(22), a larger range of α_H/α_L lowers the fraction of H-type carpoolers and raises that of L-type carpoolers, except for the L-type in the SB case. Still, NT and SB policies lead to relatively higher welfare improvements as α_H/α_L increases, and the welfare under first-best tolling is not sensitive to α_H/α_L . The relative efficiency of the second-best subsidization hence depends on whether the effect of no tolling dominates, or that of the second-best subsidization dominates. Our numerical results demonstrate that when α_H/α_L is small, the latter is stronger, and when α_H/α_L is large, the former becomes stronger. Consequently, the relative efficiency first increases and then decreases. In addition, the general ratio heterogeneity leads the relative efficiency to decrease more than the separate ratio heterogeneity, because the heterogeneous β_H and β_L make drivers benefit more from time-varying tolling.

With more heterogeneity in β_H/β_L , in Fig. 4(b), proportional heterogeneity leads to a decreasing relative efficiency of the second-best subsidization. All policies lead to relatively higher social welfare improvement with a larger β_H/β_L . However, different from the insensitivity to α_H/α_L , the welfare change from FB tolling now plays a key role. Consistent with the earlier discussion around Table 4, there are two arrival orders by changing proportional heterogeneity in the FB pricing: Case 1 before point A, and Case 2 after point A. For Case 1, a larger β_H/β_L raises the fraction of L-type carpoolers and

⁹ The third-best subsidization has the same welfare effects and relative efficiency as the second-best subsidization.

¹⁰ For general ratio case, the heterogeneity of β_H and β_L still exists as we keep $\beta_H = 5.21$, $\beta_L = 5.65$, $\gamma_H = 20.28$, $\gamma_L = 24.38$; for general proportional case, the heterogeneity of α_H and α_L also exists as we keep the average value of α at ϵ 10.

lowers that of H-type carpoolers, and H-type users dominate L-type users in the computation of social surplus. For Case 2, in contrast, the fractions of carpoolers of both types decrease with β_H/β_L . So the gain of first-best pricing increases with β_H/β_L , and increases more in Case 2. This explains why the kink at point A exists, and explains why the relative efficiency of second-best subsidization drops and slightly quicker from point A. Because carpoolers are always from the same type, this happens exactly at $\beta_H/\beta_L = 2$.

To investigate the effects of β_H/β_L with general proportional heterogeneity (the dotted line in Fig. 4(b)), we increase and decrease β_H/β_L , α_H/α_L , γ_H/γ_L by the same percentages, while keeping the ratio $\frac{\alpha_H}{\beta_H} = \frac{12.50}{5.21} = 2.40$, $\frac{\alpha_L}{\beta_L} = \frac{7.50}{6.25} = 1.20$, $\frac{\gamma_H}{\beta_H} = \frac{\gamma_L}{\beta_L} = \frac{20.28}{5.21} = 3.89$ constant. As β_H can now be lower than β_L , the minimal value of β_H/β_L must exceed 0.6, to make sure that the assumptions $\alpha_H > \beta_H$ and $\alpha_L > \beta_L$ remain satisfied. No tolling and first-best tolling impact the welfare in the same way as with proportional heterogeneity. However, with FB tolling, there is a peak at $\beta_H = \beta_L$. When $\beta_H < \beta_L$, the degree of heterogeneity lowers the welfare gain from tolling, as the two groups then become more similar, which of course differs from the increasing tendency for $\beta_H > \beta_L$. This leads the relative efficiency of second-best subsidization to increase for β_H/β_L to the left of B. Then a jump from point B to C happens, when the arrival order changes from case 0 to case 1 at $\beta_H = \beta_L$. From this point onward, the relative efficiency decreases smoothly until another arrival order appears at $\beta_H/\beta_L = 2$, where there is again a kink at point D. The relative efficiency of SB in Case 2 is again decreasing more strongly than in Case 1. In Appendix D, we discuss the full case in some more detail.

4.3. Heterogeneity and distributional effects

Apart from concerns over welfare gains, policy makers are also interested in the distributional effects, not in the least place because this has a strong impact on the social and political feasibility. A policy is naturally more likely to meet resistance from travelers if they are made worse off. Fig. 5 indicates that heterogeneity has a strong impact. Fig. 5(a–b) show

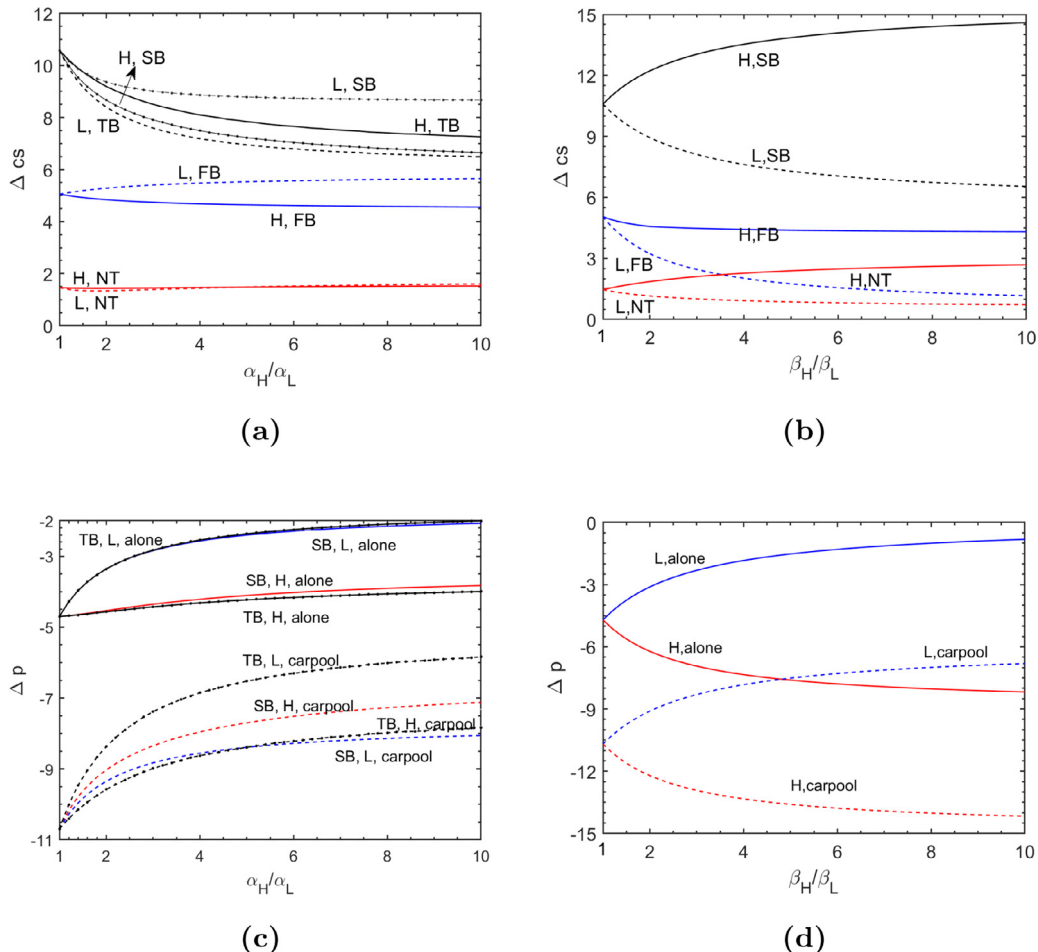


Fig. 5. Heterogeneity and distributional effects. (a) and (b) show change in consumer surplus from situation without to with carpooling; (c) and (d) show changes in generalized price from no tolling to subsidization..

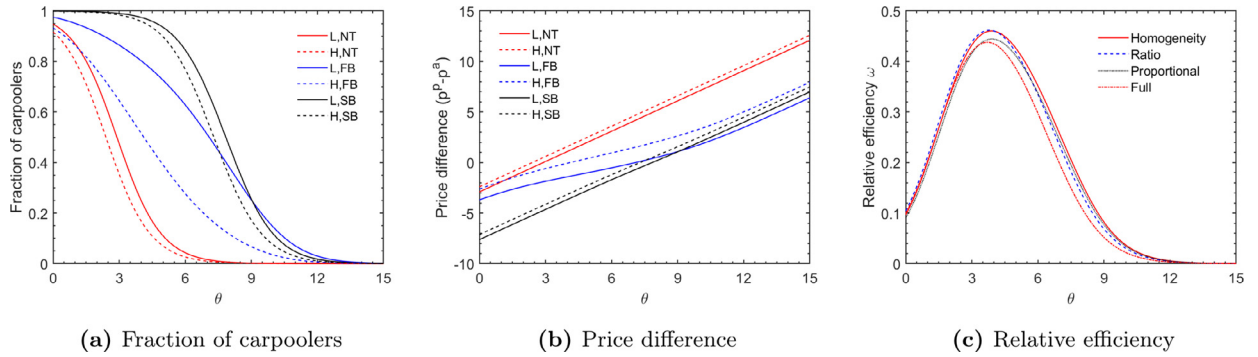


Fig. 6. Effects of the inconvenience cost θ .

the consumer surplus changes (Δcs) from the situation without carpooling to the situation with, by separately varying α_H/α_L and β_H/β_L . Fig. 5(c–d) show the generalized prices change (Δp) from no tolling to second-best subsidization.¹¹ A positive value is therefore “good” in panels (a) and (b), and “bad” in panels (c) and (d).

As Fig. 5(a–b) show that, although all users are better off by introducing carpooling, the distributional benefit under different types of heterogeneity and policies substantially differ. For all policies, heterogeneity leads the per-person consumer surplus of H type users to decrease, by raising their generalized price, and that of L type users to increase, by lowering their generalized price. The Δcs depends on whether the effects on consumer surplus with carpooling dominate over those without. In line with this, first-best tolling and carpool subsidization show different results under different types of heterogeneity. First, for the FB policy, in Fig. 5(a), L-type users benefit more, and this benefit increases with α_H/α_L for the L-type users and decreases for the H-type users. The reason is that the consumer surplus change with carpooling dominates that without carpooling for the L-type, and conversely for the H-type. Nevertheless, for proportional heterogeneity in Fig. 5(b), H-type users benefit more than L-type users, and as the two groups become more different in β_H/β_L , both of them will benefit less with an increasing β_H/β_L .

Second, we compare the second-best and third-best subsidization in Fig. 5(a). As we have discussed in Section 4.2, for the former, L-type users benefit more, and for the latter, H-type users benefit more. We can also find that as the two groups become more heterogeneous, all users benefit less, and less so for the second-best H-type users and third-best L-type users. For the second-best subsidization, it is because the optimal second-best subsidy of the H type decreases with α_H/α_L (see Eq. (23)), and that of the L type does not change. For the third-best subsidization, the undifferentiated subsidy still decreases with α_H/α_L , but for the H type, the subsidy is higher than their marginal external benefit and for the L type it is lower than their MEB. In contrast, in Fig. 5(b), H-type users benefit more than L-type users for all policies. But as β_H/β_L increases, the benefits of L-type users still decrease and those of H-type users increase, because H-type users value their time savings more.

Fig. 5(c–d) show the distributional effects of the subsidization. First, as the degree of ratio heterogeneity increases in Fig. 5(c), all users benefit less. In contrast, in increasing β_H/β_L in Fig. 5(d), L-type users benefit less from subsidization, and H-type users benefit more. The reason is that, under second-best subsidization, ratio heterogeneity raises all group's generalized prices, thus leading to a decreasing benefit for all users. Under proportional heterogeneity, although the subsidization lowers the price of L-type users and raises that of H-type users, combining with the effects of no tolling, L-type users' benefits still decrease and H-type users' still increase with β_H/β_L . Second, consistent with the earlier discussion, with ratio heterogeneity in Fig. 5(c), L-type carpoolers benefit more than H-type carpoolers. And with proportional heterogeneity in Fig. 5(d), H-type users benefit more than L-type users for both solo drivers and carpoolers. The differentiated second-best subsidy again benefits the L-type carpoolers most and the undifferentiated third-best subsidy again benefits the H-type carpoolers most (see Eqs. (23)–(25)).

4.4. Sensitivity analysis

There is little to no guidance from the literature on the values of θ and the scale of utility ϕ . Therefore, it is vital to do extensive sensitivity analyses. The effects of TT_{ff}^p and c_{fuel} are presumably similar to those for θ . Hence, these parameters will not be discussed further here.

4.4.1. Sensitivity analysis with respect to θ

In this subsection we vary the inconvenience cost of carpooling, θ , from 0 to 15. Fig. 6(a–b) show the equilibrium share of carpoolers and the price difference between driving alone and carpooling in the full setting. In Fig. 6(a), for all policies,

¹¹ In panels (a) and (b), $\Delta cs = cs(\text{with carpooling}) - cs(\text{no carpooling})$; in panels (c) and (d), $\Delta p = p(\text{with subsidy}) - c_{NT}$.

the fraction of carpoolers naturally and nonlinearly decreases with θ , consistent with the logit preferences. As expected, the share of L-type carpoolers exceeds that of the H-type. When θ increases to 12, the fraction of carpoolers under all policies goes towards 0. But the curves for NT and SB cases are steeper than that for the FB case, resulting in a relative slow declining trend to 0 with FB tolling as θ increases.

The numbers of carpoolers and solo drivers are determined by the price difference between driving alone and carpooling. Fig. 6(b) shows this price difference. It can be seen that the price difference for H-type users exceeds that for L-type users, which indicates that the L-type drivers benefit more from switching to carpooling. The curves of NT and SB both increase with the same slope 1, reflecting that θ does not affect the marginal external benefit and hence the subsidy. The intercepts are of course different, due to the subsidy provided by the SB policy. For the FB case, the price difference is not only determined by θ but also by the tolling reduction. When θ is small, the number of solo drivers N^a is almost 0 so that the price difference under FB tolling is close to that under NT case. When θ is large, almost all users will choose to drive alone, and N^a is almost N , so that the price difference in FB case is close to that in SB case. For θ between 0 and 12, the price difference of FB tolling shows a relatively gradual trend.

The inconvenience cost θ can also affect the welfare gain from a policy regime. Fig. 6(c) further shows this by giving the relative efficiency ω of the second-best subsidization under different types of heterogeneity, for varying θ . It can be observed that ω first increases and then decreases with θ , and is highest when θ is around 4. It does so for different types of heterogeneity. These patterns indicate that for intermediate values of θ , the SB subsidization is most effective. The reason is that the subsidy is relatively ineffective when carpooling is intrinsically very popular (when θ is low), or so unattractive in terms of private disutility that the marginal external benefit becomes negligible and is insufficient to induce behavior change (when θ is high).

4.4.2. Sensitivity analysis with respect to the scale of utility ϕ

Next, we look at the impact of the scale of utility ϕ , by varying the value of ϕ from 0 to 10. A larger ϕ means more deterministic preferences. Fig. 7(a) depicts the fraction of carpoolers in the full setting. When $\phi = 0$, the stochastic part of the utility function is entirely dominant, resulting in mode choices that are effectively independent of the deterministic part of utility. The probabilities then converge to 1/2, independent of the policy. When $\phi \rightarrow \infty$, the outcomes become fully deterministic, and the no tolling leads to almost 0% of carpools, and the second-best subsidization leads to almost 100% of carpoolers. However, for the first-best tolling, while the share of L-type carpoolers goes to 100%, the share for H-types goes to an interior solution, with around 40%.

Fig. 7 (b) further explains the reason, by showing the price differences between carpooling and driving alone under different policies. As ϕ increases, with no tolling, driving alone is always cheaper, and with subsidization, carpooling is always cheaper. This brings corner solutions. First-best tolling leads to different results between H-type and L-type users. For H-type users, as ϕ increases, the price difference between carpooling and driving alone becomes nearly 0, which results in an interior solution of 40%. In contrast, for L-type users, carpooling is always cheaper, thus leading to almost 100% carpoolers as the scale of utility becomes large.

Fig. 7 (c) shows the combined effects of utility scale and heterogeneity on the relative efficiency. Due to the increasing importance of idiosyncratic utility as ϕ decreases, social welfare decreases with an increasing scale of utility for all the policies. Still, an increase in the scale of utility raises the relative efficiency of the second-best subsidization for all types of heterogeneity. This is because an increase in ϕ triggers more solo drivers to go carpooling; both under FB tolling and under SB subsidization, but more so under SB case. With a stronger systematic utility, the second-best subsidization therewith becomes a more powerful instrument.

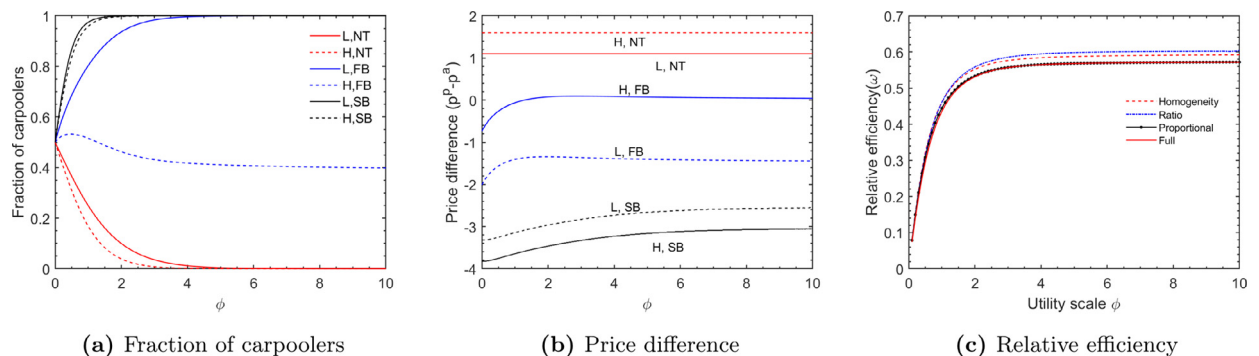


Fig. 7. Effects of the scale of utility ϕ .

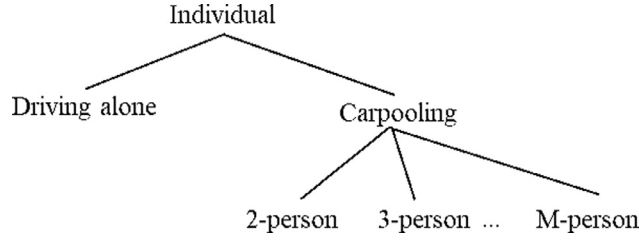


Fig. 8. Individuals choice structure.

5. Model extensions

The models we have analyzed so far investigate two-person carpooling with two user types (H and L), under ratio heterogeneity, proportional heterogeneity and the combination of these two. They illustrate the fundamental insights of carpooling with heterogeneous users. Still, there are a number of logical extensions worthy of mentioning in this section and it seems worthwhile to explore qualitatively how some of these might affect our results. The next three subsections look at three extensions, namely multiple-person carpooling, carpooling with K discrete types and heterogeneity in the value of schedule delay late (i.e. γ heterogeneity).

5.1. Model extension: Multiple-person carpooling

In our basic model, we assume carpoolers are in a two-person carpool. It is relatively straightforward to relax this assumption. To that end, we use the nested logit model (Train (2009)), where people choose between solo driving and m -person carpooling ($m = 2, 3, \dots, M$).¹² The individuals' choice structure is shown in Fig. 8. This utility structure reflects that carpooling with different numbers of partners are closer substitutes, than the choice between solo driving and carpooling as such. In interpreting the model, the reader should bear in mind that the nested logit model is meant to reflect differences in substitutability; not to suggest a dynamic sequence of decisions. The deterministic parts of the inconvenience costs, gathering times and fuel costs of m -person carpooling are assumed to be increasing with the number of person in the car, i.e. $\partial\theta[m]/\partial m > 0$, $\partial TT_{ff}^p[m]/\partial m > 0$, and $\partial c_{fuel}[m]/\partial m > 0$. The systematic part of the generalized travel cost per person per trip then becomes:

$$\begin{aligned}
 c_i^a[t] &= \max(-\beta_i t, \gamma_i t) + \alpha_i TT_i^a[t] + c_{fuel}, & \text{driving alone;} \\
 c_i^m[t, m] &= \max(-\beta_i t, \gamma_i t) + \alpha_i TT_i^m[t] + \alpha_i TT_{ff}^p[m] + \theta[m] + c_{fuel}[m], & m - \text{person carpooling.} \\
 i &= H, L; m = 2, 3, \dots, M.
 \end{aligned} \tag{31}$$

Under the random utility framework, when the idiosyncratic preferences are i.i.d. Gumbel distributed, the mode choice probabilities are given by the nested logit formula. We investigate the marginal external benefits of each type i user by switching from solo driving to m -person carpooling (MEB_i^m) and to carpooling (MEB_i^p). Compared to the two-person carpooling model, now the subsidy could also be set based on the number of persons in the car, by maximizing social welfare. We therefore consider four subsidy schemes: differentiating the subsidy over with heterogeneity and with the number of persons in the car (S_i^m), differentiating the subsidy only with heterogeneity (S_i), differentiating the subsidy only with the number of person in the car (S^m), and providing the same subsidy per carpool for all carpoolers (S^p).

Using the same approach as in the basic model, we find that the main insights from the earliest model still hold.¹³ We will mainly present the results with separate ratio and proportional heterogeneity. When taking the combination of ratio and proportional heterogeneity into consideration, the result is again the combination of these two separate effects.

We summarize the marginal external benefits of type i users by switching from solo driving to m -person carpooling and to carpooling in Table 4, and the optimal subsidies in Table 5. The marginal external benefits are similar to the MEBs in the two-person carpool case, which are again heterogeneous for ratio heterogeneity and homogeneous for proportional heterogeneity. The similarity is of course intuitive in that a two-person carpool is one special case of m -person carpool. The difference is that, for the marginal external benefits by switching from solo driving to m -person carpooling, MEB_i^m , we insert $(1 - 1/m)$ to replace $1/2$. This reflects that in the basic model, two-person carpool can reduce the congestion by $1/2$, now a marginal solo driver who switches to m -person carpooling can reduce the congestion by $1/m$. In terms of the marginal external benefit by switching to carpooling, MEB_i^p , we insert $(1 - \sum_{m=2}^M F_i^{m(p)}[m]/m)$, $i = H, L$, to replace $1/2$. The reason is that now a marginal solo driver who switches to carpooling will in expected terms reduce congestion by $F_i^{m(p)}[m]/m$, where $F_i^{m(p)}[m]$ is the conditional probability of choosing m -person carpooling (given the choice for carpooling).

¹² In our model, the variable m is considered as an integer parameter. Due to the non-linear travel cost, the results for a model with a continuous m denoting only the average number of carpoolers will be different from those from a model with a structural specification.

¹³ The derivation in this section is omitted because of length and is available upon request.

Table 4MEBs with multiple-person carpooling (per-person per trip; $i = H, L$; $m = 2, 3, \dots, M$).

	MEB_i^m	MEB_i^p
Homogeneity	$MEB^m = (1 - \frac{1}{m}) \frac{\delta N}{s}$	$MEB^p = (1 - \sum_{m=2}^M \frac{F^{m(p)}[m]}{m}) \frac{\delta N}{s}$
Ratio heterogeneity	$MEB_H^m = (1 - \frac{1}{m})(\frac{\delta N_H}{s} + \frac{\alpha_L}{\alpha_H} \frac{\delta N_L}{s})$ $MEB_L^m = (1 - \frac{1}{m}) \frac{\delta N}{s}$	$MEB_H^p = (1 - \sum_{m=2}^M \frac{F_H^{m(p)}[m]}{m})(\frac{\delta N_H}{s} + \frac{\alpha_L}{\alpha_H} \frac{\delta N_L}{s})$ $MEB_L^p = (1 - \sum_{m=2}^M \frac{F_L^{m(p)}[m]}{m}) \frac{\delta N}{s}$
Proportional heterogeneity	$MEB_H^m = MEB_L^m$ $= (1 - \frac{1}{m})(\frac{\delta_H N_H}{s} + \frac{\delta_L N_L}{s})$	$MEB_H^p = MEB_L^p$ $= (1 - \sum_{m=2}^M \frac{F_H^{m(p)}[m]}{m})(\frac{\delta_H N_H}{s} + \frac{\delta_L N_L}{s})$

Table 5Optimal subsidy with multiple-person carpooling (per-person per-trip; $i = H, L$; $m = 2, 3, \dots, M$).

	S_i^m	S^m	S_i^p	S^p
Homogeneity	-	MEB^m	-	MEB^p
Ratio heterogeneity	MEB_i^m	$\sum_{i=H,L} \sum_{j=2}^M \lambda_{ij}^j \cdot MEB_i^j$ #	MEB_i^p	$\sum_{i=H,L} \lambda_i \cdot MEB_i^p$ *
Proportional heterogeneity	MEB_H^m	MEB_H^m	MEB_H^p	MEB_H^p

#The λ_{ij}^j is determined by $\sum_{j=2}^M (\frac{\partial N_H^j}{\partial S_m} + \frac{\partial N_L^j}{\partial S_m}) = \delta(N_H + \frac{\alpha_L}{\alpha_H} N_L)/s \cdot (\frac{\partial N_H^p}{\partial S_m} + \sum_{j=2}^M \frac{1}{j} \frac{\partial N_H^j}{\partial S_m}) + \frac{\delta N}{s} \cdot (\frac{\partial N_L^p}{\partial S_m} + \sum_{j=2}^M \frac{1}{j} \frac{\partial N_L^j}{\partial S_m})$.

* $\lambda_H = \frac{\partial F_H^p}{\partial S} N_H / (\frac{\partial F_H^p}{\partial S} N_H + \frac{\partial F_L^p}{\partial S} N_L)$, $\lambda_L = \frac{\partial F_L^p}{\partial S} N_L / (\frac{\partial F_H^p}{\partial S} N_H + \frac{\partial F_L^p}{\partial S} N_L) = 1 - \lambda_H$.

The optimal subsidy again equals the marginal external benefit (MEB) or the weighted averages of the MEB's, depending on the subsidy schemes. As we consider the per-person subsidy, when the subsidy is differentiated over both the carpool forms and heterogeneity, the per-car subsidy of a certain type equals the benefit of the number of cars removed by this type: e.g., with ratio heterogeneous users, 3-person carpool of L type users saves 2 cars at a benefit of $2\delta N/s$. If the types of users cannot be observed, the per-car subsidy of m -person carpooling is the weighted average benefit of the number of cars removed by different types (i.e. mS^m).

5.2. Model extension: with K discrete types

In the basic model, we assume the users are separated into two discrete groups. We now relax this assumption and extend the basic model to K discrete groups. We keep the assumptions in Section 3.3 that, each type has a different ratio $\mu_i = \alpha_i/\beta_i$, and different values of β_i and γ_i . All groups have a common ratio $\eta_i = \gamma_i/\beta_i = \eta$. We only consider the two-person carpooling. The extension to multiple-person carpooling is conceptually straightforward, by using the same approach in Section 5.1.

To characterize the no-toll equilibrium, indices j are defined such that μ_j increases with j ; group J (with $J = K$) has the highest value. In equilibrium, carpoolers and solo drivers travel jointly, and travelers arrive ordered by μ , with the lowest values arriving closest to t^* . Generalizing for K groups results in the following generalized travel cost:

$$c_i^a = \frac{\delta_i}{s} \cdot \left(\sum_{j=1}^i \left(N_j^a + \frac{N_j^p}{2} \right) + \sum_{j=i+1}^K \left(N_j^a + \frac{N_j^p}{2} \right) \cdot \frac{\mu_i}{\mu_j} \right) + c_{fuel} \quad \forall i = 1, 2, \dots, K.$$

$$c_i^p = \frac{\delta_i}{s} \cdot \left(\sum_{j=1}^i \left(N_j^a + \frac{N_j^p}{2} \right) + \sum_{j=i+1}^K \left(N_j^a + \frac{N_j^p}{2} \right) \cdot \frac{\mu_i}{\mu_j} \right) + \alpha_i TT_{ff}^p + c_{fuel}/2 + \theta \quad \forall i = 1, 2, \dots, K. \quad (32)$$

By taking the derivatives of type- i users' travel cost with respect to the number of solo drivers and carpoolers of type j users, we obtain the following congestion effect:

$$\frac{\partial c_i^a}{\partial N_j^a} = \frac{\partial c_i^p}{\partial N_j^a} = \begin{cases} \frac{\delta_i}{s} & \text{if } j \leq i \\ \frac{\mu_i}{\mu_j} \frac{\delta_i}{s} & \text{if } j > i \end{cases}, \quad \frac{\partial c_i^a}{\partial N_j^p} = \frac{\partial c_i^p}{\partial N_j^p} = \begin{cases} \frac{\delta_i}{2s} & \text{if } j \leq i \\ \frac{\mu_i}{\mu_j} \frac{\delta_i}{2s} & \text{if } j > i \end{cases} \quad (33)$$

The marginal external benefit of type i users by switching to carpooling can now be generalized as:

$$MEB_i = \sum_{j=1}^i \frac{\delta_j N_j}{2s} \cdot \frac{\mu_j}{\mu_i} + \sum_{j=i+1}^K \frac{\delta_j N_j}{2s}. \quad (34)$$

With multiple discrete types of users, the marginal external benefits for users with low value of μ_i exceed that with high value of μ_i . The reason is that the congestion cost that a type j user imposes on a type i user depends on $1/\mu_j$, users with a given μ_i hence benefit from replacing users with a higher μ_j by users with an even higher μ_k . Conversely, a user with a certain μ_i does not suffer from replacing some users with a lower μ_j by users with an even lower μ_k . It is this asymmetry that causes the marginal external benefits to go down with an increase in the degree of heterogeneity of μ .

By maximizing the social welfare, the second-best subsidy is again the marginal external benefit of each type, i.e. $S_i^* = MEB_i$. The undifferentiated third-best subsidy can be generalized as:

$$S^* = \frac{\sum_{i=1}^K \frac{\partial F_i^p}{\partial S} \cdot N_i \cdot MEB_i}{\sum_{i=1}^K \frac{\partial F_i^p}{\partial S} \cdot N_i}. \quad (35)$$

Eq. (35) implies that when the subsidy cannot be differentiated, the third-best subsidy is still the weighted average of the MEBs, where the weights depend on the demand sensitivity of the number of carpoolers of each type to the subsidy.

5.3. Model extension: γ heterogeneity

In this subsection, we allow the cost of late arrival γ to differ across travelers, while the other values are the same for all drivers (Arnott et al. (1988, 1994), van den Berg (2014)). We again illustrate with the two-person carpooling model with K discrete types, as the extension to multiple-person carpooling is conceptually straightforward. Then when the assumption on the homogeneous η_i in Section 5.2 is relaxed, the result becomes the combination of these three types of heterogeneity (ratio heterogeneity, proportional heterogeneity and γ heterogeneity).

In the no-toll equilibrium, the ratio $\gamma_i/\beta = \eta_i$ ($i = 1, 2, \dots, K$) matters. We use i^* to denote the indifferent type between early and late arrival. Travelers with a high η_i above η_{i^*} arrive before t^* , and those with a low η_i arrive after t^* . Solo drivers and carpoolers travel jointly. Given the distribution of the early and late arrivals, we can obtain the no-toll generalized travel costs:

$$c_i^a = \begin{cases} \frac{\beta}{s} \cdot \left(\sum_{j=i^*}^K (N_j^a + N_j^p/2) \right) + c_{fuel} & \text{if } i \geq i^*, \\ \frac{\beta}{s} \cdot \left(\sum_{j=i^*}^K (N_j^a + N_j^p/2) - \sum_{j=i}^{i^*} (\eta_j - \eta_i) \cdot (N_j^a + N_j^p/2) \right) + c_{fuel} & \text{if } i < i^*. \end{cases} \quad (36)$$

$$c_i^p = \begin{cases} \frac{\beta}{s} \cdot \left(\sum_{j=i^*}^K (N_j^a + N_j^p/2) \right) + \alpha_i TT_{ff}^p + \theta + c_{fuel}/2 & \text{if } i \geq i^*, \\ \frac{\beta}{s} \cdot \left(\sum_{j=i^*}^K (N_j^a + N_j^p/2) - \sum_{j=i}^{i^*} (\eta_j - \eta_i) \cdot (N_j^a + N_j^p/2) \right) + \alpha_i TT_{ff}^p + \theta + c_{fuel}/2 & \text{if } i < i^*. \end{cases} \quad (37)$$

The marginal external benefit of type i users by switching to carpooling now becomes:

$$MEB_i = \begin{cases} \frac{\beta N}{2s} & \text{if } i \geq i^*, \\ \frac{\beta}{2s} \sum_{k=1}^{i^*} N_k \cdot (\eta_i - \eta_k) & \text{if } i < i^*. \end{cases} \quad (38)$$

With γ heterogeneity, the marginal external benefit is homogeneous for users with values of η_i above the threshold η_{i^*} , which are types that all arrive before t^* , and is heterogeneous for those with η_i below η_{i^*} (who do arrive after t^*). The reason is that the congestion cost a type j user imposes on type i user depends on the difference $\eta_j - \eta_i$, and hence users with a low value of η_i , below η_{i^*} , benefit from replacing users with a lower η_j by users with an even lower η_k . Reversely, users with a low value of η_i , below η_{i^*} , are not impacted by users who have a high value of η_i , above η_{i^*} . In addition, users with a high value of η_i above η_{i^*} do not benefit or suffer from any replacing.

In the first-best equilibrium, the values of schedule delay early and late matter. In addition to the separate γ heterogeneity shown in Arnott et al. (1988, 1993) and van den Berg (2014), carpooling also results in heterogeneity in the value of schedule delay early, due to the doubled people in the car. The result proves to be a combination of the separate γ heterogeneity and proportional heterogeneity. For the carpool subsidization, the optimal subsidy still equals the marginal external benefit or the weighted averages of the MEB's.

6. Conclusion

We have investigated the effects of carpooling in a dynamic equilibrium model of congestion, that captures various dimensions of users heterogeneity: a distribution of idiosyncratic preferences for car-pooling versus solo-driving, and heterogeneity of values of time and values of schedule delay. The share of users of carpooling is endogenous. We considered three policy scenarios: no tolling, first-best tolling, and carpool subsidization.

All commuters are better off by introducing a carpooling program, which is intuitive given its voluntary nature and the benefits it generates for non-carpoolers. Still, heterogeneity plays an important role. We investigated the marginal external benefits (MEB) of users by switching from solo driving to m -person carpooling under different types of heterogeneity. With ratio heterogeneity, the marginal external benefits for users with low value of time exceed those with high value of time. With proportional heterogeneity, the marginal external benefits are the same for all users. With γ heterogeneity, the marginal external benefits are homogeneous for users who arrive before the preferred arrival time, and heterogeneous for those who do arrive after the preferred arrival time. In terms of the marginal external benefits of switching to carpooling from solo driving, the MEBs turn out to be in expected terms, which are still heterogeneous for ratio heterogeneity and homogeneous for proportional heterogeneity.

As a large part of the benefits of carpooling goes toward the other drivers, it is worthwhile to provide a subsidy to make carpooling more attractive when no other (road pricing) policy is implemented. When the subsidy can be differentiated by the type, the optimal subsidy to carpooling (or m -person carpooling) turns out to be the marginal external benefit (MEB) of this type by switching to carpooling (or m -person carpooling). When the commuters' type cannot be observed, the weights in the weighted average undifferentiated subsidy expression depend on each type of users' demand sensitivity to the subsidy.

Using a numerical example of two-types of users, we evaluated the effects of carpool subsidization under different types of heterogeneity, both in terms of social welfare and distributional effects. The relative efficiency is the welfare increase of a policy from a base case relative to that of the first-best policy. It first increases and then decreases with the degree of ratio heterogeneity; and it always decreases with the degree of proportional heterogeneity. All users gain from second-best subsidization. But surprisingly, with ratio heterogeneity, L-type carpoolers benefit more than H-type carpoolers. With proportional heterogeneity, H-type users benefit more than L-type users for both solo drivers and carpoolers.

There are of course some limitations in the model setting. We considered a discrete setting, and we ignored alternative transport, elastic demand, and route choice. Therefore, the following possible extensions in the future study are identified. First, an elastic function for the total demand may be considered, so that the reduced travel cost may attract more commuters. Second, interactions among multiple origin-destination pairs may be modeled in a network setting. Third, public transit mode, such as a metro line, may be added to examine a multi-modes transportation system. Fourth, commuters with continuous heterogeneity may be considered. Finally, it will be interesting to study other policies to promote carpooling in our setting, such as HOV lanes and free or preferential parking for carpoolers.

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Appendix A. The marginal external benefit with ratio heterogeneity

Combining Eqs. (15)–(17) and taking the derivative of TC with respect to N_H^a , we have

$$\begin{aligned} \frac{\partial TC}{\partial N_H^a} &= c_H^a + N_H^a \frac{\partial c_H^a}{\partial N_H^a} + N_H^p \frac{\partial c_H^p}{\partial N_H^a} + N_L^a \frac{\partial c_L^a}{\partial N_H^a} + N_L^p \frac{\partial c_L^p}{\partial N_H^a} \\ &= c_H^a + \frac{\delta N_H^a}{s} + \frac{\delta N_H^p}{s} + \frac{\alpha_L}{\alpha_H} \frac{\delta N_L^a}{s} + \frac{\alpha_L}{\alpha_H} \frac{\delta N_L^p}{s} = c_H^a + \frac{\delta N_H}{s} + \frac{\alpha_L}{\alpha_H} \frac{\delta N_L}{s}. \end{aligned} \quad (A.1)$$

Similarly, we can obtain

$$\frac{\partial TC}{\partial N_H^p} = N_H^a \frac{\partial c_H^a}{\partial N_H^p} + c_H^p + N_H^p \frac{\partial c_H^p}{\partial N_H^p} + N_L^a \frac{\partial c_L^a}{\partial N_H^p} + N_L^p \frac{\partial c_L^p}{\partial N_H^p} = c_H^p + \frac{\delta N_H}{2s} + \frac{\alpha_L}{\alpha_H} \frac{\delta N_L}{2s}. \quad (A.2)$$

Substituting the above equations into Eq. (18), we can get the MEB for the H-type users:

$$MEB_H = \frac{\delta N_H}{2s} + \frac{\alpha_L}{\alpha_H} \frac{\delta N_L}{2s}. \quad (A.3)$$

Similarly, we can obtain $MEB_L = \frac{\delta N}{2s}$.

Appendix B. The second-best subsidy with ratio heterogeneity

We first calculate the optimal second-best subsidy for the H-type carpoolers. The social welfare is:

$$SW = \frac{N_H \log(e^{-\phi p_H^a} + e^{-\phi p_H^p}) + N_L \log(e^{-\phi p_L^a} + e^{-\phi p_L^p}) + N c_s^\#}{\phi} - S_L N_L^p - S_H N_H^p. \quad (B.1)$$

By taking the derivative of SW with respect to S_H through Eq. (B.1), we can obtain

$$\begin{aligned} \frac{\partial SW}{\partial S_H} &= \frac{N_H}{\phi} \frac{-\phi \left(e^{-\phi p_H^a} \frac{\partial p_H^a}{\partial S_H} + e^{-\phi p_H^p} \frac{\partial p_H^p}{\partial S_H} \right)}{e^{-\phi p_H^a} + e^{-\phi p_H^p}} + \frac{N_L}{\phi} \frac{-\phi \left(e^{-\phi p_L^a} \frac{\partial p_L^a}{\partial S_H} + e^{-\phi p_L^p} \frac{\partial p_L^p}{\partial S_H} \right)}{e^{-\phi p_L^a} + e^{-\phi p_L^p}} - N_H^p - S_H \frac{\partial N_H^p}{\partial S_H} \\ &= N_H \frac{-e^{-\phi p_H^a} \frac{\partial p_H^a}{\partial S_H} - e^{-\phi p_H^p} \frac{\partial p_H^p}{\partial S_H}}{e^{-\phi p_H^a} + e^{-\phi p_H^p}} + N_L \frac{-e^{-\phi p_L^a} \frac{\partial p_L^a}{\partial S_H} - e^{-\phi p_L^p} \frac{\partial p_L^p}{\partial S_H}}{e^{-\phi p_L^a} + e^{-\phi p_L^p}} - N_H^p - S_H \frac{\partial N_H^p}{\partial S_H}. \end{aligned} \quad (B.2)$$

By Eq. (22), we have

$$\frac{\partial N_H^a}{\partial S_H} = -\frac{\phi e^{\phi(-\alpha_H TT_{ff}^p - \theta + c_{fuel}/2 + S_H)} N_H}{(1 + e^{\phi(-\alpha_H TT_{ff}^p - \theta + c_{fuel}/2 + S_H)})^2}; \quad \frac{\partial N_H^p}{\partial S_H} = \frac{\phi e^{\phi(\alpha_H TT_{ff}^p + \theta - c_{fuel}/2 - S_H)} N_H}{(1 + e^{\phi(\alpha_H TT_{ff}^p + \theta - c_{fuel}/2 - S_H)})^2}. \quad (B.3)$$

On the other hand, from the generalized price, we obtain

$$\frac{\partial p_H^a}{\partial S_H} = \frac{\delta}{s} \left(\frac{\partial N_H^a}{\partial S_H} + \frac{1}{2} \frac{\partial N_H^p}{\partial S_H} \right) = \frac{\partial p_H^p}{\partial S_H} + 1; \quad \frac{\partial p_L^a}{\partial S_H} = \frac{\delta}{s} \frac{\alpha_L}{\alpha_H} \left(\frac{\partial N_H^a}{\partial S_H} + \frac{1}{2} \frac{\partial N_H^p}{\partial S_H} \right) = \frac{\partial p_L^p}{\partial S_H}. \quad (B.4)$$

Let $\frac{\partial SW}{\partial S_H} = 0$. Combining Eqs. (B.2)–(B.5), $\frac{\partial SW}{\partial S_H} = 0$ can then be further simplified as

$$N_H \frac{\partial p_H^a}{\partial S_H} + N_L \frac{\partial p_L^a}{\partial S_H} + S_H \frac{\partial N_H^p}{\partial S_H} = 0. \quad (B.5)$$

Substituting Eq. (B.3) and Eq. (B.4) into Eq. (B.5) yields

$$\frac{e^{\phi(-\alpha_H TT_{ff}^p - \theta + c_{fuel}/2 + S_H)} N_H}{(1 + e^{\phi(-\alpha_H TT_{ff}^p - \theta + c_{fuel}/2 + S_H)})^2} \left(-\frac{1}{2} \frac{\delta N_H}{s} - \frac{1}{2} \frac{\alpha_L}{\alpha_H} \frac{\delta N_L}{s} + S_H \right) = 0, \quad (B.6)$$

implying that $S_H^* = \frac{1}{2} \frac{\delta N_H}{s} + \frac{1}{2} \frac{\alpha_L}{\alpha_H} \frac{\delta N_L}{s}$.

Eq. (B.6) shows that $S_H < S_H^*$ yields $\frac{\partial SW}{\partial S_H} > 0$ and $S_H > S_H^*$ yields $\frac{\partial SW}{\partial S_H} < 0$. Therefore, S_H^* is the optimal subsidy that maximizes social welfare. Following the same logic, we can obtain the second-best subsidy for the L group, as expressed in Eq. (23).

Appendix C. The third-best subsidy with ratio heterogeneity

Under third-best subsidization, the subsidy for the High and Low group carpoolers is the same. Substituting $S_H = S_L = S$ into Eq. (B.1) and taking the derivative of SW with respect to S yield

$$\frac{\partial SW}{\partial S} = \frac{N_H}{\phi} \frac{-\phi \left(e^{-\phi p_H^a} \frac{\partial p_H^a}{\partial S} + e^{-\phi p_H^p} \frac{\partial p_H^p}{\partial S} \right)}{e^{-\phi p_H^a} + e^{-\phi p_H^p}} + \frac{N_L}{\phi} \frac{-\phi \left(e^{-\phi p_L^a} \frac{\partial p_L^a}{\partial S} + e^{-\phi p_L^p} \frac{\partial p_L^p}{\partial S} \right)}{e^{-\phi p_L^a} + e^{-\phi p_L^p}} - (N_L^p + N_H^p) - S \left(\frac{\partial N_L^p}{\partial S} + \frac{\partial N_H^p}{\partial S} \right) = 0. \quad (C.1)$$

Taking the derivative of the number of users and the generalized prices with respect to S , we obtain:

$$\begin{aligned} \frac{\partial N_H^a}{\partial S} &= -\frac{e^{\phi(-\alpha_H TT_{ff}^p - \theta + c_{fuel}/2 + S)} N_H}{(1 + e^{\phi(-\alpha_H TT_{ff}^p - \theta + c_{fuel}/2 + S)})^2} = -\frac{\partial N_H^p}{\partial S}; \\ \frac{\partial N_L^a}{\partial S} &= -\frac{e^{\phi(-\alpha_L TT_{ff}^p - \theta + c_{fuel}/2 + S)} N_L}{(1 + e^{\phi(-\alpha_L TT_{ff}^p - \theta + c_{fuel}/2 + S)})^2} = -\frac{\partial N_L^p}{\partial S}; \end{aligned} \quad (C.2)$$

and

$$\begin{aligned} \frac{\partial p_H^a}{\partial S} &= \frac{\delta}{s} \left(\frac{\partial N_H^a}{\partial S} + \frac{1}{2} \frac{\partial N_H^p}{\partial S} + \frac{\partial N_L^a}{\partial S} + \frac{1}{2} \frac{\partial N_L^p}{\partial S} \right); \quad \frac{\partial p_H^p}{\partial S} = \frac{\partial p_H^a}{\partial S} - 1; \\ \frac{\partial p_L^a}{\partial S} &= \frac{\delta}{s} \frac{\alpha_L}{\alpha_H} \left(\frac{\partial N_H^a}{\partial S} + \frac{1}{2} \frac{\partial N_H^p}{\partial S} \right) + \frac{\delta}{s} \left(\frac{\partial N_L^a}{\partial S} + \frac{1}{2} \frac{\partial N_L^p}{\partial S} \right); \quad \frac{\partial p_L^p}{\partial S} = \frac{\partial p_L^a}{\partial S} - 1. \end{aligned} \quad (C.3)$$

Substituting $F_H' = \frac{\partial F_H}{\partial S} = \frac{1}{N_H} \frac{\partial N_H^p}{\partial S}$, $F_L' = \frac{\partial F_L}{\partial S} = \frac{1}{N_L} \frac{\partial N_L^p}{\partial S}$ and Eqs. (C.2)–(C.3) into Eq. (C.1), and carrying out some algebraic computations, we can simplify the optimal subsidy S^* as

$$S^* = \frac{F_H' N_H}{F_H' N_H + F_L' N_L} \left(\frac{\delta N_H}{2s} + \frac{\alpha_L}{\alpha_H} \frac{\delta N_L}{2s} \right) + \frac{F_L' N_L}{F_H' N_H + F_L' N_L} \frac{\delta N}{2s}. \quad (C.4)$$

The second order condition of SW with respect to S again ensures that S^* maximizes the social welfare.

Appendix D. Full heterogeneity

All results in Section 4.3 are further confirmed in Fig. D.9. Fig. D.9(a–b) show the early arrival order in different pricing policies. Fig. D.9(c–d) show how the combination of α_H/α_L and β_H/β_L affects the social welfare and the relative efficiency of the second-best subsidization. As illustrated in Table 3, there are 2 arrival orders for NT and SB equilibrium, and 4 arrival orders for FB equilibrium. We first look at the FB equilibrium. Consistent with the earlier discussion, with varying β_H/β_L , welfare gains first decrease in Case 0 and then increase in Case 1 and Case 2, and increase more in Case 2. With varying α_H/α_L , welfare gains of FB case first increase for a small range ($1 < \frac{\alpha_H}{\alpha_L} < 1.26$) and then start to decrease for the most range. While for NT equilibrium, the change of welfare gains is clearly divided into 2 cases: when $\frac{\alpha_H}{\alpha_L} > \frac{\beta_H}{\beta_L}$, it increases with α_H/α_L , whereas when $\frac{\alpha_H}{\alpha_L} < \frac{\beta_H}{\beta_L}$, it decreases with α_H/α_L . Of course, for both cases, welfare gains of NT case decrease with β_H/β_L . Besides, although second-best subsidization has the same arrival order as NT case, the social welfare curves of SB case are not exactly downward shifted copies of the NT curves, because the subsidy provided by the government varies over the value of time and schedule delay. Depending on which welfare effects dominate, the relative efficiency of the second-best subsidization decreases with α_H/α_L , and increases with β_H/β_L when $\beta_H < \beta_L$ and starts to decrease when $\beta_H > \beta_L$. The dotted line that separates the contour plot region in Fig. D.9(d) is $\beta_H/\beta_L = \alpha_H/\alpha_L$, where the arrival order changes from the H-type in the bottleneck center to the L-type in the bottleneck center in the NT and SB case.

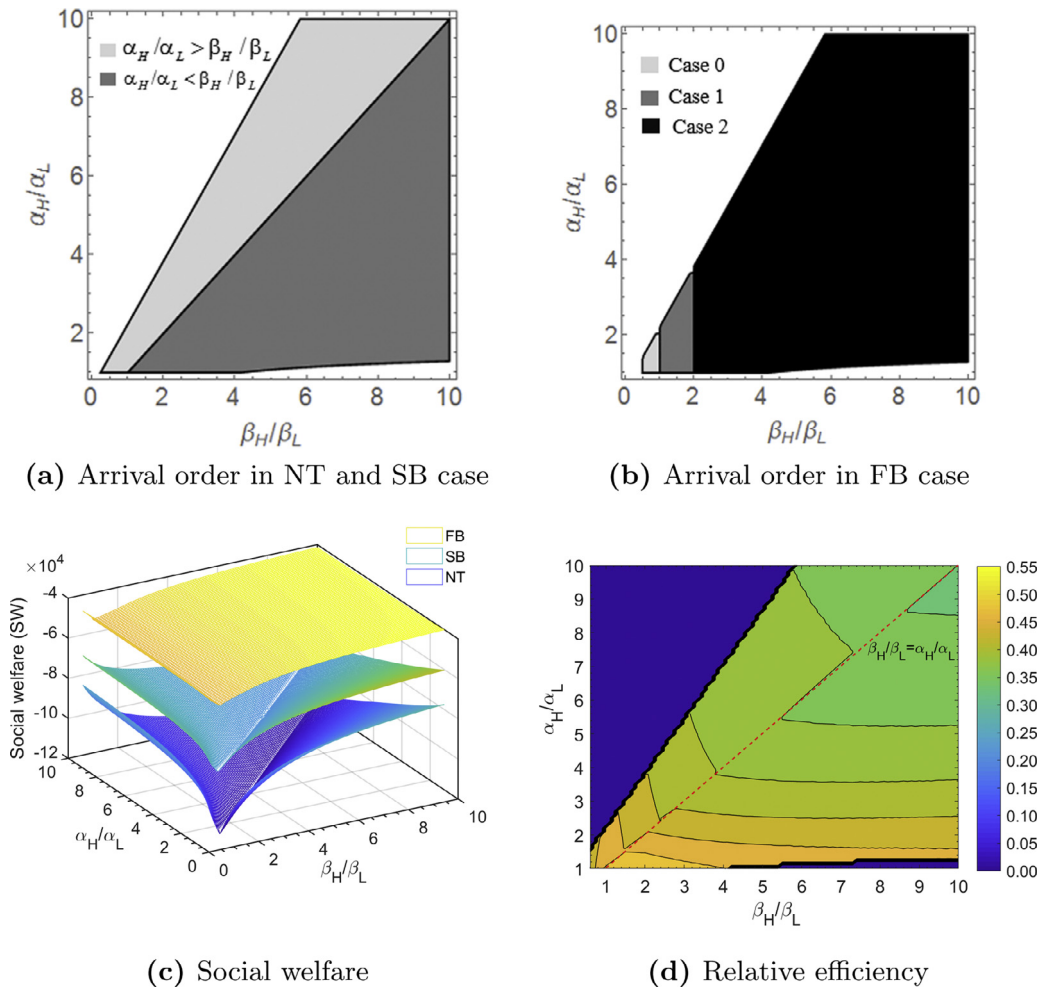


Fig. D.9. Full heterogeneity.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:<https://doi.org/10.1016/j.trb.2019.07.003>.

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